

**Los Angeles Unified School District**  
**Division of Instruction**  
**2016-2017 Statistics and Probability Curriculum Map**

## **OVERVIEW OF THE COMMON CORE MATHEMATICS CURRICULUM MAP**

### **Introduction to the Document:**

Welcome to the Los Angeles Unified School District’s Common Core Mathematics Curriculum Map. The Mathematics Curriculum Map for Los Angeles Unified School District is developed as a tool for direction and clarification. It is a living document that is interactive and web-based. There are specific, precise links to provide readily accessible resources needed to appropriately meet the rigors of the common core state standards. The curriculum map is intended to be a one-stop tool for teachers, administrators, parents, and other school support personnel. It provides information on the Common Core Standards for Mathematics, assessment sample items, and suggested instructional tools organized into units providing one easy-to-read resource.

### **Components of the Mathematics Curriculum Map:**

The curriculum map is designed around the standards for mathematics k – 12 which are divided into two sets: Practice Standards and Content standards. The Standards for Mathematical Practice are identical for each grade level. They are the expertise and understanding which the mathematics educators will seek to develop in their students. These practices are also the “processes and proficiencies” to be used as instructional “habits of mind” to be developed at all grade levels. It is critical that mathematical literacy is emphasized throughout the instructional process.

The Statistics and Probability curriculum map is grouped into five coherent units. Each unit clarifies the cluster/concept and specific standards students are to master. In addition, the relevant Mathematical Practices and learning progressions are correlated. These sections of the curriculum map define the big idea of the unit. These five units are summarized in the **Unit Organizer** which provides the overview for the year.

Instructional components are specified in:

- **Enduring Understandings** are the key understandings/big ideas that the students will learn from the unit of study. These are statements that communicate the learning in a way that engages students.
- **Essential Questions** are based on enduring understandings. They are used to gain student interest in learning and are limited in number. They promote critical or abstract thinking and have the potential of more than one “right” answer. They are connected to targeted standards and are the framework and focus for the unit.
- **Standards:** Targeted (content and skills to be taught and assessed) and supporting (content that is relevant to the unit but may not be assessed; may include connections to other content areas). This includes what students have to know and be able to do (learning targets) in order to meet the standards.

Mathematical literacy is a critical part of the instructional process, which is addressed in:

- **Key Vocabulary and Language Goals** which clearly indicate strategies for meeting the needs of EL and SEL students

Planning tools provided are:

- **Instructional Strategies** lead to enduring understandings. They are varied and rigorous instructional strategies to teach content. They are plan experiences that reinforce and enrich the unit while connecting with the standards and assessments. Instructional strategies addresses individual student needs, learner perspectives, integration of technology, learning styles, and multiple intelligences.
- **Resources and Performance Tasks** offer concept lessons, tasks, and additional activities for learning.
- **Assessments:** This is also a listing of formative and summative Assessments to guide backwards planning. Student progress in achieving targeted standards/expected learning is evaluated. Entry-level (formative)-based on summative expectations, determine starting points for learning. Benchmark-determine progress of learning, misconceptions, strengths/weaknesses along the learning trajectory.
- **Differentiation** (📖) falls into three categories:
  - **Front Loading:** strategies to make the content more accessible to all students, including EL, SEL and students with special needs. This defines prerequisite skills needed to be successful.
  - **Enrichment:** activities to extend the content for all learners, as all learners can have their thinking advanced, and to support the needs of GATE students. These are ideas to deepen the conceptual understanding for advanced learners.
  - **Intervention:** alternative methods of teaching the standards, in which all students can have a second opportunity to connect to the learning, based on their own learning style. They guide teachers to resources appropriate for students needing additional assistance

### Using the Mathematics Curriculum Map:

The guide can be thought of as a menu. It cannot be expected that one would do every lesson and activity from the instructional resources provided. To try to teach every lesson or use every activity would be like ordering everything on a menu for a single meal. It is not a logical option. Nor is it possible given the number of instructional days and the quantity of resources. That is why the document is called a "**Mathematics Curriculum Map**" and not a "**Mathematics Pacing Plan**." And, like a menu, teachers select, based on instructional data, which lessons best fit the needs of their students – sometimes students need more time with a concept and at other times, less.

An effective way to use this guide is to review and assess mathematical concepts taught in previous grades to identify potential learning gaps. From there, teachers would map out how much time they feel is needed to teach the concepts within the unit based on the data of their students' needs. For example, some classes may need more time devoted to developing expressions and equations, while another class in the same course may need more focused time on understanding the concept of functions.

The starting point for instructional planning is the standards and how they will be assessed. By first considering how the standards will be assessed, teachers can better select the instructional resources that best build mathematical understanding. There are hundreds of resources available, both publisher- and teacher-created, as well as web-based, that may be used to best teach a concept or skill. Collaborative planning, both within and among courses, is strongly encouraged in order to design effective instructional programs for students.

### **Learning Progressions:**

The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents can explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. This would be useful in teacher preparation and professional development, organizing curriculum, and writing textbooks.

### **Standards for Mathematical Practice:**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

The Mathematics Curriculum Map is a living document—it is neither set in stone for all time nor is it perfect. Teachers and other users are encouraged to provide on-going feedback as to its accuracy, usability, and content. Please go to [achieve.lausd.net](http://achieve.lausd.net) and click on the **2016-2017 Curriculum Map** link, and share your comments and suggestions. Your participation in making this instructional guide a meaningful and useful tool for all is needed and appreciated.

**T**he grade level Common Core State Standards-aligned Curriculum Maps of the courses in this 2016 edition of the CCSS *Mathematics Curriculum Map* are the result of the collective expertise of the LAUSD Secondary Mathematics Team.

The District extends its gratitude to the following Statistics and Probability curriculum map development team:

Susan Sim Kim, Firoza Kanji, Scott DePutron, April Bain, Mihran Dabagian, and Stephen Lange.

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**Statistics and Probability – UNIT 1**  
**Interpreting Categorical and Quantitative Data**

**Introduction:** Instructional time will focus on creating and interpreting visual displays of data. Students will choose appropriate visual displays, measures of center/spread for a specific data set. They will compare data sets using summary statistics.

CLUSTER	COMMON CORE STATE STANDARDS
<p>Summarize, represent, and interpret data on a single count or measurement variable.</p>	<p><u>HSS.ID.A.1</u>*            Represent data with plots on the real number line (dot plots, histograms, and box plots).</p> <p><u>HSS.ID.A.2</u>*            Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> <p><u>HSS.ID.A.3</u>*            Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p> <p><u>HSS.ID.A.4</u>            Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p>
<p>Summarize, represent, and interpret data on two categorical and quantitative variables.</p>	<p><u>HSS.ID.B.5</u>            Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p><u>HSS.ID.B.6</u>            Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p><u>HSS.ID.B.6.A</u>            Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or</p>

CLUSTER	COMMON CORE STATE STANDARDS
Interpret linear models.	<p>choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p> <p><u>HSS.ID.B.6.B</u></p> <p>Informally assess the fit of a function by plotting and analyzing residuals.</p> <p><u>HSS.ID.B.6.C</u></p> <p>Fit a linear function for a scatter plot that suggests a linear association.</p> <p><u>HSS.ID.C.7</u></p> <p>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p><u>HSS.ID.C.8</u></p> <p>Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p><u>HSS.ID.C.9</u></p> <p>Distinguish between correlation and causation.</p>
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
<ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>	<p>Statistics and Probability Progression</p> <p><a href="http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf">http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</a></p>

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>• Students create and interpret appropriate models of data (dotplots, stemplots, boxplots, histograms, two-way tables, scatterplots).</li> <li>• Students compare shape, center, and spread for distributions of quantitative data.</li> <li>• Students understand how measures of center and spread change in response to extreme observations/outliers.</li> <li>• Students standardize data using z-scores and percentiles. Students understand the Normal distribution and can use it to estimate population percentages. <i>Ex. Empirical Rule (68-95-99.7)</i></li> <li>• Students compute marginal and conditional distributions and can mathematically justify whether two categorical variables are associated.</li> <li>• Students can describe associations between two quantitative variables (form, direction, strength).</li> <li>• Students can fit a function to the data (linear, quadratic, exponential, etc.).</li> <li>• Students interpret components of a line of best fit (slope and y-intercept) and the correlation coefficient, <math>r</math>.</li> <li>• Students understand the distinction between correlation and causation.</li> </ul>	<p>How can data be expressed so that their accurate meaning is concisely presented to a specific audience?</p> <p>How can you compare two distributions of quantitative data utilizing shape, center, spread, and outliers?</p> <p>How do extreme observations affect summary statistics?</p> <p>In what ways does knowing the outcome of one event affect subsequent events?</p> <p>What are the patterns in data collected and how are they useful?</p> <p>What are the statistical tools that can be used to create models, interpret data, and make predictions?</p> <p>How can you explain the association between two categorical/quantitative variables?</p> <p>Does correlation imply causation? Explain why or why not.</p>	<p>Average</p> <p>Bar graph</p> <p>Bimodal distribution</p> <p>Box plot</p> <p>Correlation (<math>r</math>)</p> <p>Coefficient of determination (<math>r^2</math>)</p> <p>Cumulative frequency distribution</p> <p>Dotplot</p> <p>Empirical rule</p> <p>Explanatory variable</p> <p>Extrapolation</p> <p>Five-number summary</p> <p>Frequency</p> <p>Histogram</p> <p>Interpolation</p> <p>Interquartile range (IQR)</p> <p>Linear correlation</p> <p>Mean</p> <p>Median</p> <p>Mode</p> <p>Normal curves</p> <p>Normal distributions</p> <p>Outlier</p> <p>Parameter</p> <p>Percentile</p> <p>Pie chart</p> <p>Population</p> <p>Qualitative variable</p> <p>Quantitative variable</p> <p>Quartile</p> <p>Range</p> <p>Relative-frequency histogram</p> <p>Residual</p> <p>Residual plot</p> <p>Resistant measure</p> <p>Response variable</p> <p>Sample</p> <p>Scatter plot</p> <p>Skewed left/right</p>

		Slope Standard Deviation Standard Normal distribution Standard units Statistic Stemplot Symmetric distribution Uniform distribution z-score
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RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>Graphing calculator: Create graphs of quantitative data</li> <li>Statistical Software (Fathom Dynamic Software, IBM SPSS Statistics, etc.)</li> <li><a href="http://desmos.com">Desmos.com</a>: See regression tutorial</li> <li>StatKey: Free resource for creating graphs: <a href="http://lock5stat.com/statkey/">http://lock5stat.com/statkey/</a></li> <li>Microsoft Excel/Google sheets: Create bar graphs, pie charts, segmented bar graphs</li> <li>Rossmann Chance Applet collection (<a href="http://www.rossmanchance.com/applets/index.html">http://www.rossmanchance.com/applets/index.html</a>): Guess the correlation applet</li> <li><a href="https://www.amstat.org/censusatschool/">https://www.amstat.org/censusatschool/</a>: Data resource from students around the world</li> <li><i>Against All Odds</i>: <a href="https://www.learner.org/resources/series65.html">https://www.learner.org/resources/series65.html</a></li> <li><a href="http://mathbits.com/">http://mathbits.com/</a>: Tips to help with graphing calculators</li> <li>Gapminder: <a href="https://www.gapminder.org/for-teachers/">https://www.gapminder.org/for-teachers/</a></li> <li>AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. <a href="http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html">http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html</a></li> </ul>	<ul style="list-style-type: none"> <li>When asking students to describe a distribution, students must describe the shape, outliers, center, and spread (SOCS) of the distribution. The acronym SOCS may help students remember to describe each of these features.</li> <li>Often times, students forget to describe data in context of a scenario. Statistics problems are almost always about some form of real world data, which should be emphasized as the students describe SOCS.           <ul style="list-style-type: none"> <li>E.g., when describing the center of the distribution of games won by all NBA teams last year, students should mention that the center of the distribution is _____ <b>number of games won</b>, rather than solely the center is _____.</li> </ul> </li> <li>The effect of outliers on a set of data is very important in the real world.           <ul style="list-style-type: none"> <li>An example that can be used would be to provide students with 4 different test scores: 75, 80, 70, and 95. Ask the students to find the student's average test score, as well as the median, standard deviation, and Interquartile range. Then ask what would happen to each summary statistic if the student cheated on his/her next test and received a 0. This should help them</li> </ul> </li> </ul>	<p><b>Formative Assessment</b>          SBAC - <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p> <hr/> <p><b>LAUSD Interim Assessment</b>          District assessments can be accessed through:  <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a>  <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a></p> <p>Use your Single Sign On to access the Interim Assessments</p> <hr/> <p>California will be administering the Smarter Balanced Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include items from all High School Common Core strands, including Statistics and Probability. For examples, visit the Smarter Balanced Assessment at: <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	<p style="text-align: center;">understand which types of statistics are resistant to outliers.</p> <ul style="list-style-type: none"> <li>• Provide students with several different sets of data. Ask them when it would be appropriate to use each type of graphical display. Emphasize the difference between graphs for categorical vs. quantitative data. <ul style="list-style-type: none"> <li>○ Use statistical software to create a set of data and display a dotplot, boxplot, and histogram. Ask students to critique each type of graph (advantages/disadvantages) and justify their reasoning.</li> </ul> </li> <li>• The Normal distribution is applied throughout many of the inference procedures used in AP Statistics. This distribution should be addressed, but does not need to be as emphasized as much as in the AP course.</li> <li>• Help students understand the purpose of a z-score in terms of location in a distribution. <ul style="list-style-type: none"> <li>○ For example, being 10 units higher than the mean is significant if you are talking about height in centimeters, but not very significant if you are talking about a person's salary.</li> </ul> </li> <li>• To introduce two-variable quantitative data, a variety of in-class activities may be used. You may have students collect data on their hand spans and height and analyze the correlation.</li> <li>• Many of the interpretations that are required for two-variable quantitative data are challenging for students (slope, y-intercept, correlation coefficient, etc.). It may help to provide the students with sentence stems for each interpretation and have them practice interpreting the meanings in context.</li> </ul>	

## LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners

- Students will use comparative language when comparing the center and spread for distributions of data in context.

*Example:* The center for distribution 1 is **higher/lower** than the center for distribution 2. There is **more/less** variability in distribution 2 than in distribution 1.

- Students will collaboratively decide which graphical display would be best suited for a specific set of data and *justify* orally or in writing why this graphical display is preferred over another.

*Example:* I would choose (graphical display) because...

- Students will interpret the slope and y-intercept of a regression equation in context.

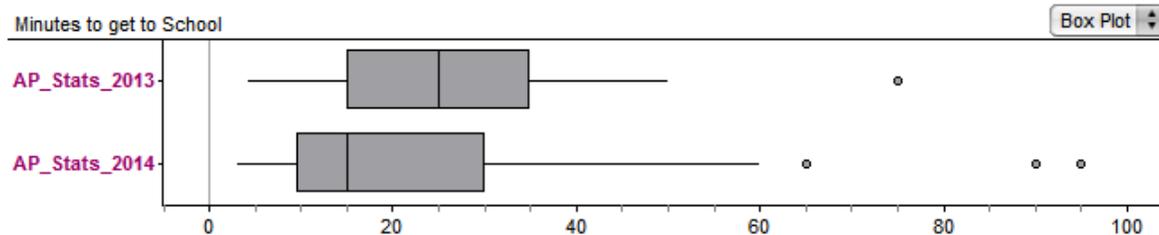
*Example:* Suppose we have a data set where  $x$  = the age of a plant in months,  $y$  = the height of a plant, and the regression equation is  $\hat{y} = 1.2 + 2.3x$ .

**Slope:** For every additional month that passes, the **predicted** height of the plant will increase by 2.3 cm.

**Y-intercept:** For a plant that is 0 months old, the **predicted** height is 1.2 cm.

## PERFORMANCE TASK

An AP Statistics teacher asked two of his previous classes the question, “How long does it take you to get to school?” Below are parallel boxplots summarizing the students’ answers to this question. Compare the time it takes to get to school for the AP Stats classes in 2013 and 2014.



When asked to describe or compare distributions of data, students will be expected to describe the **Shape, Outliers, Center, and Spread (SOCS) in context of the scenario**. Here is a sample answer to the question above.

**Shape:** The distribution of minutes to get to school for the AP stats class in 2013 is *roughly symmetrical*, while the distribution of minutes to get to school for the AP stats class in 2014 is *skewed to the right*.

**Outliers:** For the class of 2013, there was one student who took 75 minutes to get to school, while in the class of 2014, there were 3 students whose times are considered outliers (65, 90, and 95 minutes to get to school). All four of these students took unusually longer to get to school than their classmates.

\*\* The 1.5(IQR) rule can be used to test for outliers if you are given the actual data.

**Center:** The students in the class of 2013 take longer to get to school, on average, than the students in the class of 2014 (median = 25 minutes in 2013 vs. 15 in 2014).

\*\*Measures of center include the mean and median.

**Spread:** There is more variability in the distribution of minutes to get to school for the class of 2014 than for the class of 2013. Although the middle 50% of the distributions are both spread over 20 minutes (definition of IQR), the distribution of data in the class of 2014 has a larger range ( $95 - 3 = 92$  minutes) than the class of 2013 ( $75 - 4 = 71$  minutes).

\*\*Measures of spread include IQR ( $Q3 - Q1$ ), standard deviation, and range ( $\text{max} - \text{min}$ ).

**Here are some additional questions to ask your students (strictly about quantitative data):**

What was the cost of your last \_\_\_\_\_ (e.g. haircut)?

How many siblings/cousins/relatives do you have?

How many texts do you send per day? How many can you send per minute? (provide students with a list of messages and time them)

What is your GPA?

How old are you?

How many hours of sleep do you get? How long do you spend on homework/video games/watching Netflix/eating?

**\*\*Have the students create graphs of these topics and describe the distributions or compare different class sets of data using SOCS.**

DIFFERENTIATION 		
UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p><b>Statistics and Probability:</b></p> <ul style="list-style-type: none"> <li>As an opening activity, conduct an activity in which students collect a quick set of data from the class and analyze the mean, median, mode, and range. Survey questions could include: the number of letters in your name, number of cousins versus aunts and uncles, typical number of minutes to get to school, primary transportation method to get to school, shoe size.</li> <li>Model a simple activity involving collecting a quick set of data from the class and analyzing (calculate) the mean, median, mode, and range (e.g. the number of letters in your first and last name).</li> <li>Students could create a graphical display of the data collected from the activity above.</li> <li>Create a KWL (Know, Want to know, Learned) chart for unit vocabulary.</li> </ul>	<p>Acceleration for high achieving students:</p> <ul style="list-style-type: none"> <li>While teaching two-variable quantitative data, encourage students to visit the site <a href="http://www.tylervigen.com">http://www.tylervigen.com</a> to demonstrate that correlation does not imply causation. Have students research other publications that involve correlation.</li> <li>Present students with Anscombe's Quartet to help students understand the value of graphing data prior to calculating summary statistics.</li> <li>Present an example of Simpson's Paradox (2016 FRQ #6 from the AP Statistics Exam).</li> </ul>	<ul style="list-style-type: none"> <li>Students who are having difficulty understanding standard deviation may need to be introduced to the idea of mean average deviation (MAD) first. This may help students understand the idea of average variability from the mean more clearly.</li> <li>Many of the interpretations that are required for two-variable quantitative data are challenging for students (slope, y-intercept, correlation coefficient, etc.). It may help to provide the students with sentence stems for each interpretation and have them practice interpreting the meanings in context.</li> </ul>

**Statistics and Probability – UNIT 2**  
**Designing and Evaluating Studies**

**Introduction:** Instructional time should focus on designing and evaluating statistical studies. Students will learn the differences between sample surveys, observational studies, and experiments. Students will learn the consequences of bias and how to construct a study to minimize bias. In Unit 5, students will make inferences and conclusions based on the types of studies that they have learned in this unit.

CLUSTER	COMMON CORE STATE STANDARDS
<p>Understand and evaluate statistical studies: sample surveys, experiments, and observational studies.</p> <p>Justify conclusions from statistical studies.</p>	<p><u><a href="#">HSS.IC.A.1</a></u>            Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p> <p><u><a href="#">HSS.IC.B.3</a></u>            Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p> <p><u><a href="#">HSS.IC.B.6</a></u>            Evaluate reports based on data.</p>
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
<ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> </ol>	<p>Statistics and Probability Progression  <a href="http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf">http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</a></p>

7. Look for and make use of structure.	
8. Look for and express regularity in repeated reasoning.	

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>Students distinguish a population/parameter from a sample/statistic.</li> <li>Students recognize the purpose for different random sampling methods (simple random sample, stratified, cluster, etc.).</li> <li>Students compare and contrast sample surveys, observational studies, and experiments.</li> <li>Students understand different experimental designs such as blocking and matched pairs.</li> </ul>	<ul style="list-style-type: none"> <li>What is the difference between a statistic and a parameter?</li> <li>What is the best sampling method given scenario? Justify your response.</li> <li>Which types of studies allow you to conclude a causal relationship between two variables and why?</li> <li>What are the key components of good experimental design?</li> </ul>	Bias Blocking Census Cluster sampling Confounding Convenience sampling Double-blind experiment Individuals Nonresponse Placebo effect Population data Randomized experiment Sample size ( $n$ ) Simple random sample Simulation Stratified sampling Systematic sampling Variable Voluntary Response

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>Graphing calculator: <i>randint</i>( command for random sampling</li> <li><a href="http://www.random.org">Random.org</a>: Create random digits</li> <li>The Data and Story Library (DASL): <a href="http://lib.stat.cmu.edu/DASL">http://lib.stat.cmu.edu/DASL</a></li> <li><i>Against All Odds</i>: <a href="https://www.learner.org/resources/series65.html">https://www.learner.org/resources/series65.html</a></li> <li>AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. <a href="http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html">http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html</a></li> </ul>	<ul style="list-style-type: none"> <li>Emphasize that students should not just refer to a sampling method as biased. Students must be able to explain why a sampling method is biased and whether it will lead to an <i>overestimation</i> or <i>underestimation</i> of the parameter of interest.</li> <li>Show students multiple methods for creating a simple random sample (SRS): putting slips of paper in a hat and mixing it, using dice/coins, using a table, and using technology (calculator or <a href="http://www.random.org">Random.org</a>).</li> <li>Emphasize the importance of random sampling. Have students create a graphic organizer</li> </ul>	<p><b>Formative Assessment</b></p> <p>SBAC – <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p> <hr/> <p><b>LAUSD Periodic Assessment</b></p> <p>District assessments can be accessed through:  <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a>  <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a></p> <p>Use your Single Sign On to access the Interim Assessments</p>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	<p>comparing an SRS, stratified random sample, and cluster sample. They should compare the advantages and disadvantages of each.</p> <ul style="list-style-type: none"> <li>You may choose to have the students design an experiment prior to learning about the key principles of experimental design. For example, ask them to design an experiment to determine if yoga reduces stress using 100 volunteers. You will be surprised to see how many students will incorporate the ideas of comparative groups, randomization, control, and replication prior to formally learning these concepts.</li> <li>Have students write a proposal in which they will apply the full statistical investigative process. This could be part of a year-long project.</li> </ul>	<p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include items from all High School Common Core strands, including Statistics and Probability. For examples, visit the SMARTER Balance Assessment at: <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p>

**LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners**

- Students will critique studies that have been published.
 

*Example:* Since this survey was posted online, it was subject to voluntary response bias. This could lead to an over/under-estimation of...
- Students will explain orally or in writing how to incorporate randomization into a study.
 

*Example:* Obtain a list of the names of the 100 subjects and assign each subject a number from 1-100. Use a random number generator to select 50 different numbers. The subjects who correspond to those 50 numbers will be assigned to the treatment group, and the remaining 50 subjects will be assigned to the control group.
- Students will design experiments and clearly explain orally or in writing how all principles of experimental design are incorporated.
 

*Example:* In order to determine if listening to music while studying helps improve achievement, first we will randomly assign... We will maintain control by...

**PERFORMANCE TASK**

Does ginkgo improve memory? The law allows marketers of herbs and other natural substances to make health claims that are not supported by evidence. Brands of ginkgo extract claim to improve memory and concentration. A randomized comparative experiment found no statistically significant evidence for such effects. The subjects were 230 healthy volunteers over 60 years old. They were randomly assigned to ginkgo or a placebo pill (a dummy pill that looks and tastes the same). All the subjects took a battery of tests for learning and memory before treatment started and again after six weeks. (Moore, Basic Practice of Statistics, 5e, 2009)

- (a) The study was double-blind. What does this mean?
- (b) Comment briefly on the extent to which results of this study can be generalized to some larger population, and the extent to which cause and effect has been established.
- (c) Explain why it is more advantageous to use 230 volunteers in this study, rather than 30.
- (d) Using the random digits below (starting at line 103), choose the first four members of the ginkgo group. Explain your method.

<b>103</b>	45467	71709	77558	00095	32863	29485	82226	90056
<b>104</b>	52711	38889	93074	60227	40011	85848	48767	52573
<b>105</b>	95592	94007	69971	91481	60779	53791	17297	59335
<b>106</b>	68417	35013	15529	72765	85089	57067	50211	47487

**DIFFERENTIATION **

UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p><b>Statistics and Probability:</b></p> <ul style="list-style-type: none"> <li>• As an opening activity, guide students in the design and execution of an in class experiment (e.g. taste test of different types of bottled water - determine a question of interest, what variables to collect, who are the observational units, how is it random). Students use "common sense" to design an experiment, then the teacher can connect experiment vocabulary throughout the unit back to students' original ideas).</li> <li>• Statistical problem solving is an investigative process that involves four components: formulate questions, collect</li> </ul>	<p>Acceleration for high achieving students:</p> <p>Encourage students to find studies that have incorporated different forms of random sampling, including a multi-stage design. Ask them to explain why they think that the people who planned the study chose each method.</p> <p>Help students distinguish stratified sampling from cluster sampling: in a stratified sample, the population is divided into strata and sample "some from all;" whereas in a cluster sample, we divide the population into clusters and sample "all from some."</p>	<ul style="list-style-type: none"> <li>• Make a foldable for students to write down all of the important vocabulary in the unit, along with their definitions/applications. Another strategy to build understanding of vocabulary would be to use a word wall in the classroom or have the students create a word puzzle.</li> <li>• Provide graphic organizers for survey and experimental design.</li> </ul>

**DIFFERENTIATION** 

UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p>data, analyze data, and interpret results. The connection should be reviewed prior to detailing all of the principles of a good experimental design.</p> <ul style="list-style-type: none"> <li>• Make the connection between hypotheses and conclusions in science classes and the current unit.</li> </ul>	<p>As a group project, have students design and conduct an experiment to investigate the effects of response bias in surveys. Allow them to choose the specific topic, but ensure that their topic can answer at least one of the following questions (adopted from Josh Tabor):</p> <ul style="list-style-type: none"> <li>• Can the wording of a question create response bias?</li> <li>• Does providing additional information create response bias?</li> <li>• Do the characteristics of the interviewer create response bias?</li> <li>• Does anonymity change the responses to sensitive questions?</li> <li>• Does manipulating the answer choices/order of answer choices change the response?</li> <li>• Can revealing other peoples' answers to a question create response bias?</li> </ul>	

**Statistics and Probability – UNIT 3**  
**Probability Rules**

**Introduction:** Instructional time is spent on the concepts of randomness and probability. Students will learn about the Law of Large Numbers and how to calculate the likelihood of random events and outcomes. Students will learn about the counting principle, permutations, and combinations prior to learning how to apply the basic rules of probability.

CLUSTER	COMMON CORE STATE STANDARDS
Use the rules of probability to compute probabilities of compound events.	<p><u>HSS.CP.A.1</u> Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").</p> <p><u>HSS.CP.B.7</u> Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model.</p> <p><u>HSS.CP.B.8</u> (+) Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model.</p> <p><u>HSS.CP.B.9</u> (+) Use permutations and combinations to compute probabilities of compound events and solve problems.</p>
Understand independence and conditional probability and use them to interpret data.	<p><u>HSS.CP.A.2</u> Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p> <p><u>HSS.CP.A.3</u> Understand the conditional probability of <math>A</math> given <math>B</math> as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of <math>A</math> and <math>B</math> as saying that the conditional probability of <math>A</math> given <math>B</math> is the same as the probability of <math>A</math>, and the conditional probability of <math>B</math> given <math>A</math> is the same as the probability of <math>B</math>.</p> <p><u>HSS.CP.A.4</u> Construct and interpret two-way frequency tables of data when two categories are associated with each object being</p>

CLUSTER	COMMON CORE STATE STANDARDS
	<p>classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p><u>HSS.CP.A.5</u></p> <p>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p> <p><u>HSS.CP.B.6</u></p> <p>Find the conditional probability of <math>A</math> given <math>B</math> as the fraction of <math>B</math>'s outcomes that also belong to <math>A</math>, and interpret the answer in terms of the model.</p>
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
<ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>	<p>Statistics and Probability Progression  <a href="http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf">http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</a></p>

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>Students understand the concept of probability and the Law of Large Numbers.</li> <li>Students interpret probability as a long-run relative frequency.</li> <li>Students apply the counting principle, permutations, and combinations to solve problems.</li> <li>Students understand when and how to apply the basic probability rules.</li> <li>Students understand conditional probability and independence.</li> </ul>	<ul style="list-style-type: none"> <li>How do you apply probability rules?</li> <li>How do you define the sample space for a random phenomenon?</li> <li>What is independence and how do you determine whether given events are independent?</li> <li>What is the difference between permutations and combinations?</li> <li>How do you compute probabilities for mutually exclusive (disjoint) or overlapping events?</li> <li>How do you compute conditional probabilities?</li> </ul>	Addition rule of probability Combination Complement Conditional probability Disjoint/Mutually exclusive events Event Factorial (!) Fundamental counting principle Independent events Law of large numbers Multiplication rule of probability Permutation Sample space

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>Graphing calculators: factorials (!), nPr, cPr</li> <li>Graphic Organizer such as Venn Diagram, tree diagram, two-way tables, etc.</li> <li>Geogebra applets for simulating randomness</li> <li>**resource for combinations/permutations</li> <li><i>Against All Odds:</i> <a href="https://www.learner.org/resources/series65.html">https://www.learner.org/resources/series65.html</a></li> <li>AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. <a href="http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html">http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html</a></li> </ul>	<ul style="list-style-type: none"> <li>Help students use Venn diagrams, tree diagram, or two-way tables to visualize probability calculations.</li> <li>When introducing the idea of conditional probability, display the entire two-way table and then physically cover up the rows or columns that are not part of the “given.” This will help students focus on only the row or column that represents the event that we know has occurred.</li> <li>Use Venn Diagrams and formulas to demonstrate the difference between mutually exclusive/disjoint events and independent events.</li> </ul>	<p><b>Formative Assessment</b></p> <p>SBAC - <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p> <hr/> <p><b>LAUSD Periodic Assessment</b></p> <p>District assessments can be accessed through:  <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a>  <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a></p> <p>Use your Single Sign On to access the Interim Assessments</p> <hr/> <p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include items from all High School Common Core strands, including Statistics and Probability. For examples, visit the SMARTER Balance Assessment at:  <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p>

**LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners**

- Students will justify verbally or in writing whether two events are independent.

*Example Stem:* Justify whether \_\_\_\_ and \_\_\_\_ are independent.

- Students will comprehend a written scenario and be able to apply the appropriate probability rules.

*Example Stem:* Given the probability of \_\_\_\_\_, what is the probability of \_\_\_\_\_.

- Students will apply the general multiplication rule and interpret the answer.

*Example:* Find the probability of choosing two face cards from a standard deck without replacement and interpret what this probability means.

**PERFORMANCE TASK**

A psychologist is interested in the relationship between handedness (left or right) and IQ scores. He collected the following data from a random sample of 259 high school students.

	<55	55-<70	70-<85	85-<100	100-<115	115-<130	130-<145	>145
Left	1	2	6	25	32	15	14	3
Right	4	7	15	44	51	23	15	2

1. What is the probability that a student from this group has an IQ greater than 130?
2. What is the probability that a student has an IQ greater than 130, given that she is left-handed?

**DIFFERENTIATION **

UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p><b>Statistics and Probability:</b></p> <ul style="list-style-type: none"> <li>• As an opening activity, allow students to play a game of chance. For example, Two Dice Sum game (<a href="http://www.mathwire.com/data/dicetoss2.html">http://www.mathwire.com/data/dicetoss2.html</a>). Students will develop a strategy for winning that can later be quantified with probabilities.</li> <li>• Clearly define the difference between a sample and a population. This is a very important idea for students to understand early in the unit and throughout the rest of</li> </ul>	<p>Acceleration for high achieving students:</p> <ul style="list-style-type: none"> <li>• Students could use Bayes’s theorem to solve probability questions that require “backward” in a tree diagram.</li> <li>• Present the birthday problem: Ask students to predict the probability that any two people in the class share the same birthday. Then ask them to justify their predictions using probabilities.</li> <li>• The Monte Hall problem: Discuss how probability can be applied to making decisions.</li> </ul>	<ul style="list-style-type: none"> <li>• Students will make layered books to learn key vocabularies related to probabilities.</li> <li>• Students will learn about using graphic organizers and understand the meaning of mutually exclusive/disjoint.</li> <li>• Students need to understand the conversion between fractions and decimals.</li> </ul>

DIFFERENTIATION 

UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p>the course.</p> <ul style="list-style-type: none"> <li>• Review components of combinations and permutations, specifically factorials.</li> <li>• Understand the meaning of statistics and probability.</li> <li>• Understand vocabulary words such as sample space, events, etc.</li> <li>• Students should have basic knowledge of fractions that they learned in previous math classes and make connection to the basic rule of probability.</li> </ul> $P(A) = \frac{\text{Number of Outcomes}}{\text{Total Number of Outcomes}}$		

**Statistics and Probability – UNIT 4**  
**Using Probability to Make Decisions**

Introduction: Instructional time will be spent on applying probability rules to create a probability distribution model with a main focus on discrete data sets. Students will use these probability distribution models to make appropriate decisions.

CLUSTER	COMMON CORE STATE STANDARDS
<p>Calculate expected values and use them to solve problems.</p>	<p><u>HSS.MD.A.1</u>            (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.</p> <p><u>HSS.MD.A.2</u>            (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.</p> <p><u>HSS.MD.A.3</u>            (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.  <i>For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</i></p> <p><u>HSS.MD.A.4</u>            (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. <i>For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?</i></p>
<p>Use probability to evaluate outcomes of decisions.</p>	<p><u>HSS.MD.B.5</u>            (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.</p> <p><u>HSS.MD.B.5.A</u>            Find the expected payoff for a game of chance. <i>For example, find the expected winnings from a state lottery ticket or a</i></p>

CLUSTER	COMMON CORE STATE STANDARDS
	<p><i>game at a fast-food restaurant.</i></p> <p><u>HSS.MD.B.5.B</u></p> <p>Evaluate and compare strategies on the basis of expected values. <i>For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</i></p> <p><u>HSS.MD.B.6</u></p> <p>(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p> <p><u>HSS.MD.B.7</u></p> <p>(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
<ol style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with mathematics.</li> <li>5. Use appropriate tools strategically.</li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>	<p>Statistics and Probability Progression</p> <p><a href="http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf">http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</a></p>

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>Students learn about discrete random variables and their probabilities distributions.</li> <li>Students calculate and interpret the mean (expected value) of a discrete random variable.</li> <li>Students understand that mean is the expected value and calculate expected value.</li> </ul>	<ul style="list-style-type: none"> <li>How can you identify and distinguish between discrete and continuous random variables?</li> <li>How do you interpret the probability of a continuous random variable as the area under a density curve?</li> <li>How do you compute the mean (expected value) of a discrete random variable from its probability distribution?</li> </ul>	<p>Expected value Independent trials Mean of a probability distribution Probability distribution Random continuous variable Random discrete variable</p>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. <a href="http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html">http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html</a></li> <li>StatKey: Free resource for creating simulations <a href="http://lock5stat.com/statkey/">http://lock5stat.com/statkey/</a></li> <li>Create your own theoretical and experimental probability spinner: <a href="https://illuminations.nctm.org/adjustablespinner/">https://illuminations.nctm.org/adjustablespinner/</a></li> </ul>	<ul style="list-style-type: none"> <li>Remind students about the general definition of a distribution: a list of the possible values a variable can take and how often it takes those values.</li> <li>Have students interpret the expected value as a long-run average.</li> <li>Illustrate the difference between discrete and continuous random variables. <ul style="list-style-type: none"> <li>For example, a person's foot length is continuous while a person's shoe size is discrete.</li> </ul> </li> <li>Emphasize that the graphical display for a discrete random variable is a histogram while the graphical display for a continuous random variable is a density curve. Relate this back to the graphical displays learned in Unit 1.</li> <li>Remind students that they already learned how to calculate probabilities under a Normal curve in Unit 1, which is a continuous distribution.</li> </ul>	<p><b>Formative Assessment</b> SBAC - <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p> <hr/> <p><b>LAUSD Periodic Assessment</b> District assessments can be accessed through: <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a> <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a></p> <p>Use your Single Sign On to access the Interim Assessments</p> <hr/> <p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include items from all High School Common Core strands, including Statistics and Probability. For examples, visit the SMARTER Balance Assessment at: <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a>.</p>

**LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners**

- Students will exchange ideas to determine how to calculate probabilities of events involving random variables.

*Example:* I think we should apply \_\_\_\_ principle in order to determine the correct probability.

- Students will identify and interpret the expected value of a random variable.

*Example:* The expected value, \_\_\_\_\_, of \_\_\_\_\_ (random variable) is the long-run average value of X, distributed over a very large number of trials.

- Students defend their decision by quantifying the probability and explaining in context.

*Example:* If I only guessed on a multiple-choice exam with \_\_\_\_ questions with \_\_\_\_ options, I would only have \_\_\_\_ probability of passing the exam.

**PERFORMANCE TASK**

In Roulette, 18 of the 38 spaces on the wheel are black. Suppose you decide to try your luck and bet \$1 on black on the next 10 spins of a roulette wheel (recall that if you win, you win \$1, and if you lose, you lose \$1). Let  $X$  = the number of times you hit black in 10 spins.

For each question, provide a mathematical path to your answer.

- Find  $P(\text{you win } \$2)$ .
- Find  $P(\text{you lose money})$ .
- Find  $P(\text{you win money})$ .

**DIFFERENTIATION **

UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p><b>Statistics and Probability:</b></p> <ul style="list-style-type: none"> <li>• Use a simulation activity to demonstrate the difference between an experimental and a theoretical probability distribution.                             <ul style="list-style-type: none"> <li>○ The website below allows you to create your own spinner.  <a href="https://illuminations.nctm.org/adjustablesprinter/">https://illuminations.nctm.org/adjustablesprinter/</a></li> </ul> </li> <li>• Review how to calculate a weighted average and make the connection to the expected value of a discrete random variable.</li> </ul>	<p>Acceleration for high achieving students:</p> <ul style="list-style-type: none"> <li>• Teach the binomial and geometric distributions and how to calculate probabilities using these distributions.</li> <li>• Teach students how to determine whether the conditions for the Normal approximation to a binomial distribution are met.</li> <li>• Show students how to calculate the mean (expected value) and standard deviation for sums and differences of independent random variables.</li> </ul>	<ul style="list-style-type: none"> <li>• Re-emphasize the definition of probability and the basic probability rules.</li> <li>• Emphasize the idea of a sample space and how listing all possible outcomes may be helpful in creating a probability distribution.                             <ul style="list-style-type: none"> <li>○ e.g. Show students how to create a sample space when rolling two different number cubes.</li> </ul> </li> </ul>

**Statistics and Probability – UNIT 5**  
**Making Inferences and Justifying Conclusions**

Introduction: Instructional time will be spent on making inferences and justifying conclusions based on simulations. Students will simulate sampling distributions to estimate a population proportion or mean and develop the concept of margin of error. Students will use these simulated sampling distributions to make decisions whether differences between parameters are significant.

CLUSTER	COMMON CORE STATE STANDARDS
<p>Make inferences and justify conclusions from statistical studies.</p>	<p><u><a href="#">HSS.IC.B.4</a></u>            Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p> <p><u><a href="#">HSS.IC.B.5</a></u>            Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p>
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
<p>1. Make sense of problems and persevere in solving them.</p> <p>2. Reason abstractly and quantitatively.</p> <p>3. Construct viable arguments and critique the reasoning of others.</p> <p>4. Model with mathematics.</p> <p>5. Use appropriate tools strategically.</p> <p>6. Attend to precision.</p> <p>7. Look for and make use of structure.</p> <p>8. Look for and express regularity in repeated reasoning.</p>	<p>Statistics and Probability Progression  <a href="http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf">http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf</a></p>

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>Students understand simulation of random behavior and how to simulate a sampling distribution.</li> <li>Students understand the Central Limit Theorem.</li> <li>Students understand confidence intervals and critical values.</li> <li>Students construct and interpret a confidence interval for a population proportion or a population mean.</li> <li>Students learn how to calculate and interpret the margin of error.</li> <li>Students learn to estimate a population proportion and mean.</li> <li>Students learn to write hypotheses and make decisions based on the results of a simulation.</li> <li>Students learn to interpret Type I and Type II errors in context, and give the consequences of each.</li> </ul>	<ul style="list-style-type: none"> <li>What is the purpose of a confidence interval?</li> <li>How does the Central Limit Theorem apply to sampling distributions?</li> <li>How do you construct and interpret a confidence interval?</li> <li>What are the factors that affect the margin of error of a confidence interval?</li> <li>How do you estimate a population proportion or a population mean?</li> <li>What is the difference between a null and alternate hypothesis?</li> <li>How do you interpret a simulation in order to make a decision about a hypothesis?</li> <li>What is statistical significance?</li> <li>After making a conclusion, what types of error could be made?</li> <li>How do you describe Type I and Type II errors in context?</li> </ul>	<p>Alternate hypothesis  Central Limit Theorem  Confidence interval  Confidence level  Critical values  Hypothesis testing  Margin of error  Null hypothesis  P-value  Sample proportion  Sample test statistics  Sampling distributions  Standard error of the mean  Statistical significance  Type I error  Type II error</p>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>Confidence interval applet: <a href="http://bcs.whfreeman.com/tps5e/">http://bcs.whfreeman.com/tps5e/</a></li> <li>StatKey (<a href="http://lock5stat.com/statkey">lock5stat.com/statkey</a>): Create sampling distributions to analyze confidence intervals and p-values more efficiently</li> <li><i>Statistical Reasoning in Sports</i>, Tabor, J &amp; Franklin, C.</li> <li><i>Workshop Statistics: Discovery with Data</i>, 4th ed., Rossman, A &amp; Chance, B.</li> <li><i>Statistics Through Applications</i>, 2nd ed., Starnes, Yates, &amp; Moore.</li> <li>What Is Happening in This Class? (WIHC): Student questionnaire <a href="http://www.chemeng.ntua.gr/courses/ped2/files/WIHC.pdf">http://www.chemeng.ntua.gr/courses/ped2/files/WIHC.pdf</a></li> </ul>	<ul style="list-style-type: none"> <li>Incorporate various activities into this unit. Since the students will be making inferences only through a simulation-based approach, be sure to allow the students to work with data in a hands-on manner as often as possible. For example, you may have the students use coins, dice, cards, or spinners to create an approximate sampling distribution. Then, you may use technology (Fathom, Statkey, etc.) to re-create a sampling distribution with a large number of trials. Students will be more likely to understand the idea of a p-value and a margin of error after participating in numerous hands-on simulations in class.</li> <li>When writing hypotheses, emphasize that the null hypothesis is usually a statement of “no</li> </ul>	<p><b>Formative Assessment</b>  SBAC - <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p> <hr/> <p><b>LAUSD Periodic Assessment</b>  District assessments can be accessed through:  <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a>  <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a></p> <p>Use your Single Sign On to access the Interim Assessments</p> <hr/> <p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include</p>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
<ul style="list-style-type: none"> <li>• <i>Against All Odds:</i> <a href="https://www.learner.org/resources/series65.html">https://www.learner.org/resources/series65.html</a></li> <li>• Rossman Chance Applet collection (<a href="http://www.rossmanchance.com/applets/index.html">http://www.rossmanchance.com/applets/index.html</a>): Reeses Pieces Applet for sampling distributions</li> <li>• AP Stats Monkey: This site includes a wonderful collection of resources written by teachers and collected by Jason Molesky. <a href="http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html">http://apstatsmonkey.com/StatsMonkey/Statsmonkey.html</a></li> </ul>	<p>effect” or “no difference.” Also, emphasize that the hypotheses are always stated in terms of a parameter, not in terms of the statistic that is collected.</p> <ul style="list-style-type: none"> <li>• A single estimate of a population parameter is called a “point estimate” since it represents a single point on the number line.</li> <li>• Help students understand that the margin of error in a confidence interval accounts for variability due only to random selection or random assignment; it does not compensate for any bias in the data collection process.</li> <li>• Help students make connections between the formulas in this unit and previously learned formulas (mean, standard deviation, z-score, etc.).</li> <li>• Provide students with sentence frames to help them interpret confidence intervals and confidence level.</li> <li>• When students write the conclusion to a simulation question, emphasize that the answer is always an approximation.</li> </ul>	<p>items from all High School Common Core strands, including Statistics and Probability. For examples, visit the SMARTER Balance Assessment at: <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p>

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will discuss and interpret confidence intervals for a population proportion or a population mean.

*Example:* The confidence interval for \_\_\_\_\_ is (\_\_\_\_, \_\_\_\_). Interpret this interval in context.

- Students will write all inference conclusions using the language of the original question of interest.

*Example Stem:* Since the probability of the observed result or more extreme occurring is **high/low**, we **have/do not have** convincing evidence to conclude that \_\_\_\_\_ (alternative hypothesis in context) is true.

- Students will use data from a randomized experiment to compare, in writing, two treatments.

*Example:* Treatment A was more effective than treatment B because...

- Students will read closely to determine the appropriate inference procedure for a given scenario.

*Example:* I would choose \_\_\_\_\_ (inference procedure) because...

PERFORMANCE TASK

1. A survey was conducted involving 250 out of 125,000 families living in a city. The average amount of income tax paid per family in the sample was \$3540 with a standard deviation of \$1150. Construct and interpret a 99% confidence interval for the total taxes paid by all the families in the city.
2. During an angiogram, heart problems can be examined via a small tube (a catheter) threaded into the heart from a vein in the patient's leg. It is important that the company that manufactures the catheter maintain a diameter of 2.00 millimeters (mm). Each day, quality control personnel make several measurements to ensure that the diameter has not changed. If they discover a problem that causes the diameter to be different than 2.00 mm, they will stop the manufacturing process until it is corrected. Suppose on one specific day, quality control personnel take a random sample of 35 measurements and find the mean to be 2.009 mm and the standard deviation to be 0.021 mm.
  - a) Find the standard error of the sample mean for samples of size 35. Interpret this value in context.
  - b) Do these data give convincing evidence that the company should stop the manufacturing process? Use a significance test with  $\alpha = 0.05$  to find out.
  - c) Calculate a 95% confidence interval for  $\mu$ . Does your interval support your decision from part (b)?
3. Provide students with a published survey or have them collect data ( $n \geq 30$ ). Analyze the data to estimate a population mean and develop a margin of error.

DIFFERENTIATION 		
UDL/ FRONT LOADING	ACCELERATION	INTERVENTION
<p><b>Statistics and Probability:</b></p> <ul style="list-style-type: none"> <li>As an opening activity to help demonstrate the concept of sampling distributions is to take a large population of pennies and have the students take SRSs of size 1, 5, and 20 and find the sample mean year for each sample. Have the students plot the distributions on three different chart papers. This should also help demonstrate the Central Limit Theorem.</li> <li>Review the differences between a sample and a population, and a statistic and a parameter.</li> </ul>	<p>Acceleration for high achieving students:</p> <ul style="list-style-type: none"> <li>Students further their study in inference by learning specific hypothesis tests (z-test, t-test) and how to utilize a graphing calculator to run each test.</li> <li>Discuss inferences for categorical data (chi-square) and for the slope of a regression line.</li> <li>Discuss different transformations to achieve linearity (power, log, etc.).</li> </ul>	<ul style="list-style-type: none"> <li>Remind students that the sampling distribution of a statistic tells us the possible values of the statistic and how likely they are to occur.</li> <li>Remind students that parameters are fixed, and that there is nothing “special” about one confidence interval (i.e. the one sample that was taken may not even capture the parameter).</li> <li>Emphasize that, when conducting a hypothesis tests, an outcome that would rarely occur if a claim were true is good evidence that the claim is false. This may help students better understand the idea of a significance level.</li> </ul>

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