Los Angeles Unified School District
Office of Curriculum Instruction and School Support
2014-2015 Honors Advanced Mathematics Curriculum Map

Overview of the Common Core Mathematics Curriculum Map

Introduction to the Document:

Welcome to the Los Angeles Unified School District's Common Core Mathematics Curriculum Map. The Mathematics Curriculum Map for Los Angeles Unified School District is developed as a tool for direction and clarification. It is a living document that is interactive and web-based. There are specific, precise links to provide readily accessible resources needed to appropriately meet the rigors of the common core state standards. The curriculum map is intended to be a one-stop tool for teachers, administrators, parents, and other school support personnel. It provides information on the Common Core Standards for Mathematics, assessment sample items, and suggested instructional tools organized into units providing one easy-to-read resource.

Components of the Mathematics Curriculum Map:

The curriculum map is designed around the standards for mathematics k – 12 which are divided into two sets: Practice Standards and Content standards. The Standards for Mathematical Practice are identical for each grade level. They are the expertise and understanding which the mathematics educators will seek to develop in their students. These practices are also the "processes and proficiencies" to be used as instructional "habits of mind" to be developed at all grade levels. It is critical that mathematical literacy is emphasized throughout the instructional process.

The Honors Advanced Mathematics curriculum map is grouped into six coherent units. Each unit clarifies the cluster/concept and specific standards students are to master. In addition, the relevant Mathematical Practices and learning progressions are correlated. These sections of the curriculum map define the big idea of the unit. These six units are summarized in the **Unit Organizer** which provides the overview for the year.

Instructional components are specified in:

- **Enduring Understandings** are the key understandings/big ideas that the students will learn from the unit of study. These are statements that communicate the learning in a way that engages students.
- **Essential Questions** are based on enduring understandings. They are used to gain student interest in learning and are limited in number. They promote critical or abstract thinking and have the potential of more than one "right" answer. They are connected to targeted standards and are the framework and focus for the unit.
- **Standards**: Targeted (content and skills to be taught and assessed) and supporting (content that is relevant to the unit but may not be assessed; may include connections to other content areas). This includes what students have to know and be able to do (learning targets) in order to meet the standards.

Mathematical literacy is a critical part of the instructional process, which is addressed in:

- **Key Vocabulary** and **Language Goals** which clearly indicate strategies for meeting the needs of EL and SEL students

Planning tools provided are:

- **Instructional Strategies** lead to enduring understandings. They are varied and rigorous instructional strategies to teach content. They are plan experiences that reinforce and enrich the unit while connecting with the standards and assessments. Instructional strategies addresses individual student needs, learner perspectives, integration of technology, learning styles, and multiple intelligences.
- **Resources** and **Performance Tasks** offer concept lessons, tasks, and additional activities for learning.
- **Assessments:** This is also a listing of formative and summative Assessments to guide backwards planning. Student progress in achieving targeted standards/expected learning is evaluated. Entry-level (formative)-based on summative expectations, determine starting points for learning. Benchmark-determine progress of learning, misconceptions, strengths/weaknesses along the learning trajectory.
- **Differentiation** (🕮) falls into three categories:
    - **Front Loading:** strategies to make the content more accessible to all students, including EL, SEL and students with special needs. This defines prerequisite skills needed to be successful.
    - **Enrichment:** activities to extend the content for all learners, as all learners can have their thinking advanced, and to support the needs of GATE students. These are ideas to deepen the conceptual understanding for advanced learners.
    - **Intervention:** alternative methods of teaching the standards, in which all students can have a second opportunity to connect to the learning, based on their own learning style. They guide teachers to resources appropriate for students needing additional assistance

Using the Mathematics Curriculum Map:

The guide can be thought of as a menu. It cannot be expected that one would do every lesson and activity from the instructional resources provided. To try to teach every lesson or use every activity would be like ordering everything on a menu for a single meal. It is not a logical option. Nor is it possible given the number of instructional days and the quantity of resources. That is why the document is called a "***Mathematics Curriculum Map***" and not a "*Mathematics Pacing Plan*." And, like a menu, teachers select, based on instructional data, which lessons best fit the needs of their students – sometimes students need more time with a concept and at other times, less.

An effective way to use this guide is to review and assess mathematical concepts taught in previous grades to identify potential learning gaps. From there, teachers would map out how much time they feel is needed to teach the concepts within the unit based on the data of their students' needs. For example, some classes may need more time devoted to developing expressions and equations, while another class in the same course may need more focused time on understanding the concept of functions.

The starting point for instructional planning is the standards and how they will be assessed. By first considering how the standards will be assessed, teachers can better select the instructional resources that best build mathematical understanding. There are hundreds of resources available, both publisher- and teacher-created, as well as web-based, that may be used to best teach a concept or skill. Collaborative planning, both within and among courses, is strongly encouraged in order to design effective instructional programs for students.

Learning Progressions:

The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents can explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. This would be useful in teacher preparation and professional development, organizing curriculum, and writing textbooks.

Standards for Mathematical Practice:

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

The MIG is a living document—it is neither set in stone for all time nor is it perfect. Teachers and other users are encouraged to provide on-going feedback as to its accuracy, usability, and content. Please go to math.lausd.net and click on the **2014-2015 Curriculum Map** link, and share your comments and suggestions. Your participation in making this instructional guide a meaningful and useful tool for all is needed and appreciated.

The grade level Common Core State Standards-aligned Curriculum Maps of the courses in this 2014 edition of the *CCSS Mathematics Curriculum Map* are the result of the collective expertise of the LAUSD Secondary Mathematics Team.

The District extends its gratitude to the following Honors Advanced curriculum map development team:

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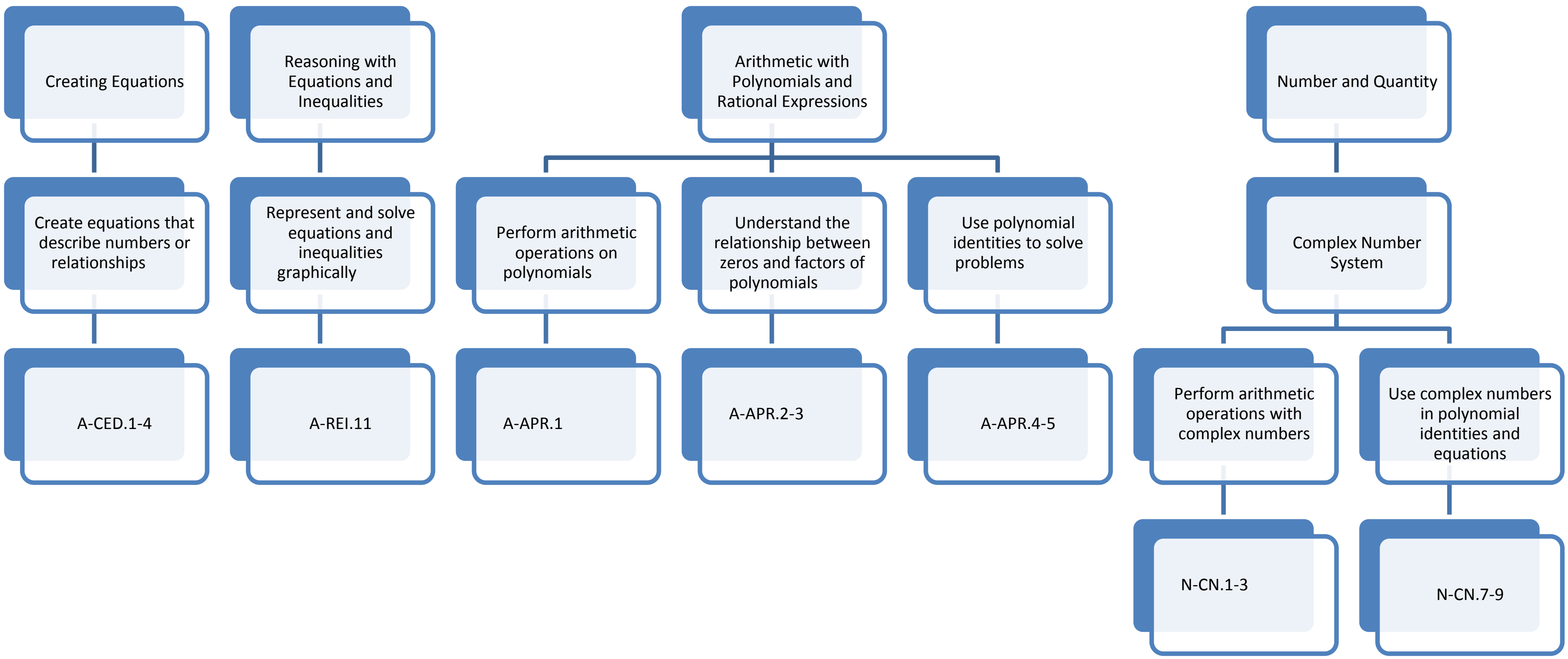
Honors Advanced Mathematics

Unit 1

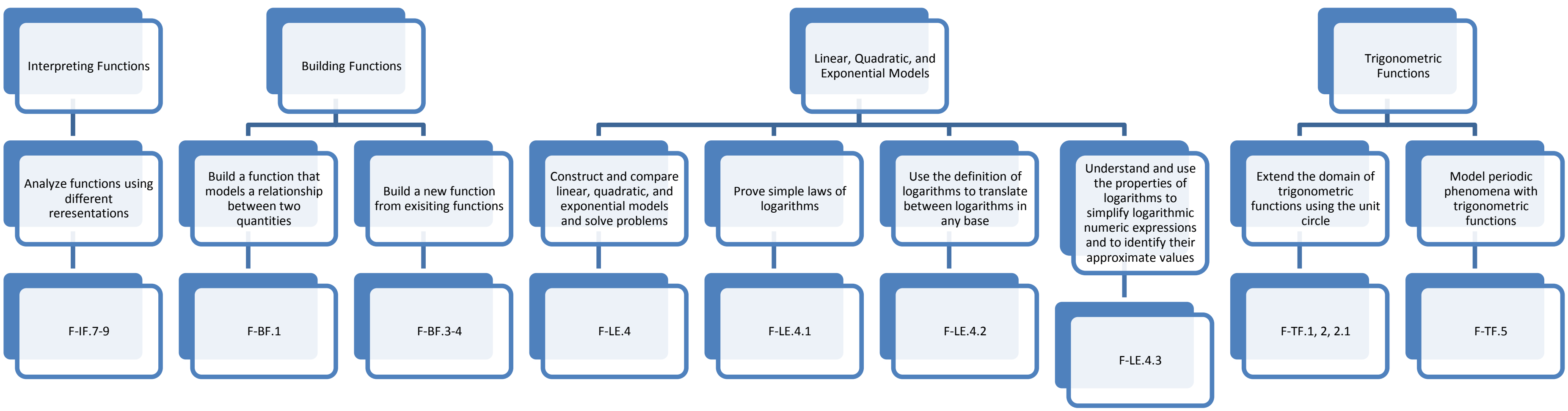
Introduction and Preliminaries to Advanced Mathematics



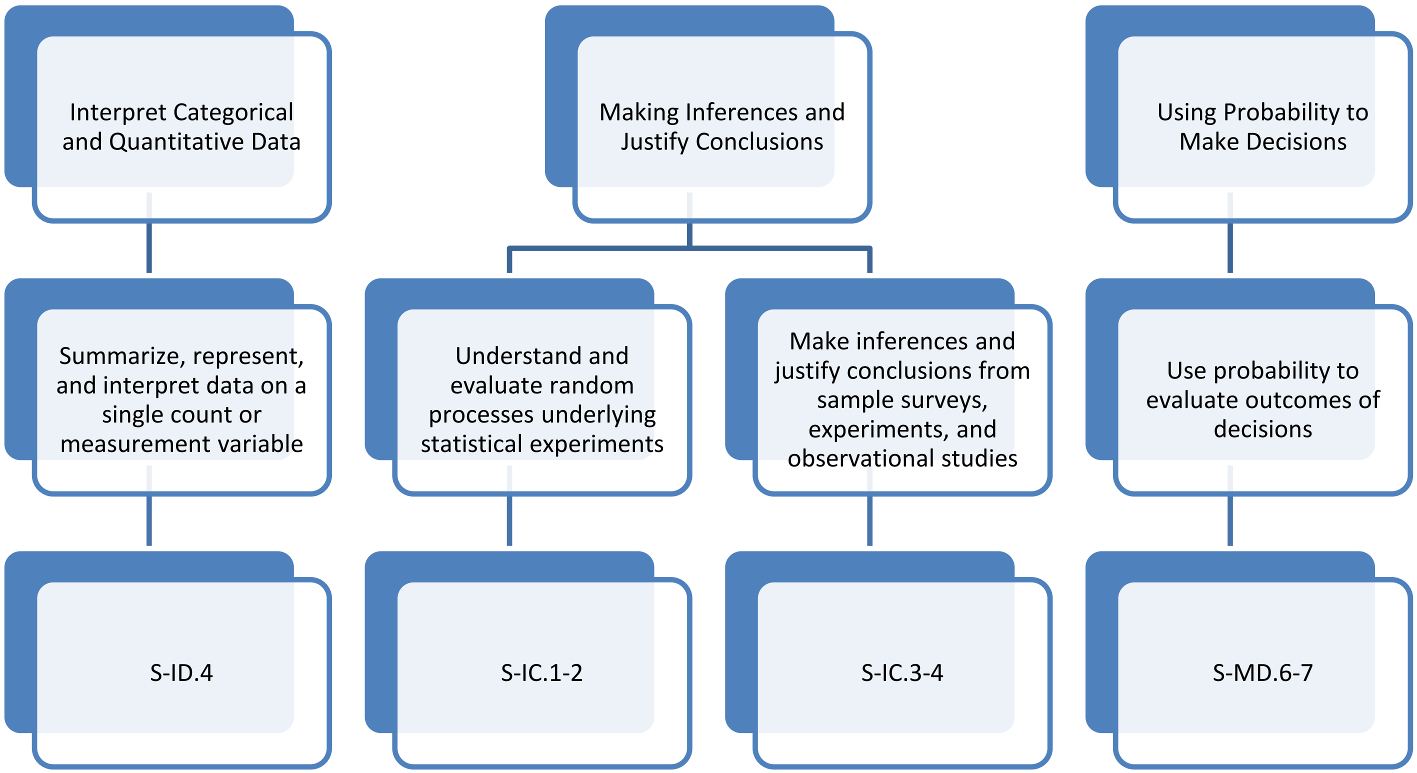
Honors Advanced Mathematics

Unit 2

Functions and Trigonometry



Honors Advanced Mathematics
Unit 3
Statistics and Probability



Honors Advanced Mathematics

Unit 4

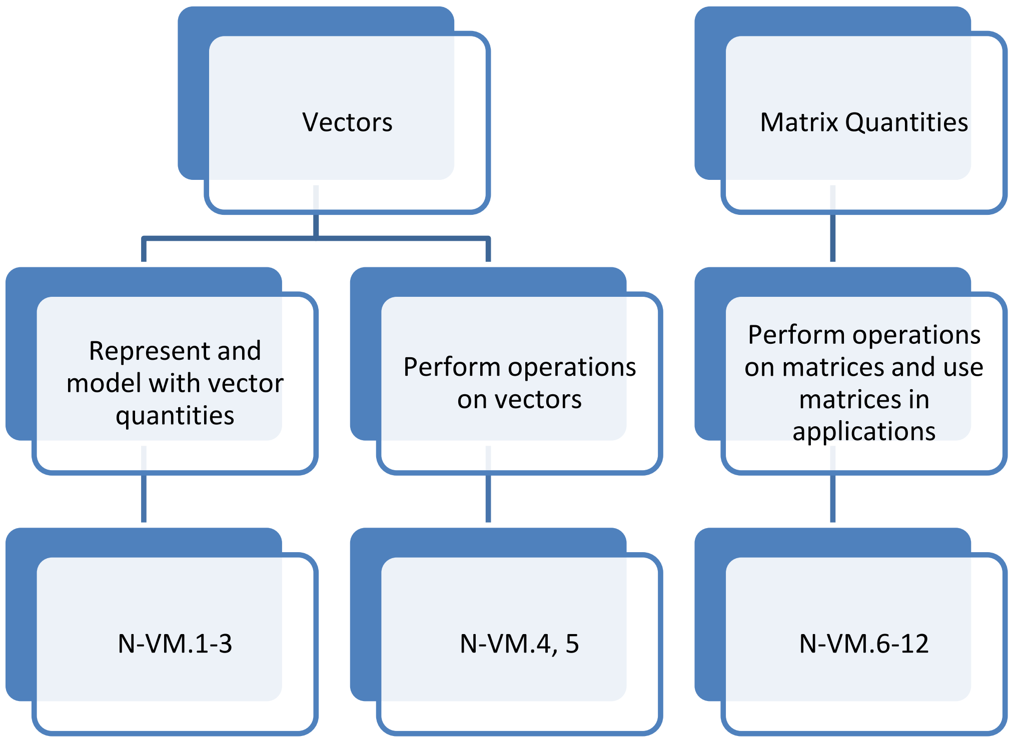
Polar Equations and the Complex Plane

- Conics
  - Translate between the geometric description and equation for a conic section
    - G-GPE.3, 3.1
- Complex Number System
  - Represent complex numbers and their operations on the complex plane
    - N-CN.4-6
- Polar Equations
  - Parametric and Polar Function
    - N-CN.4-5
  - Demonstrate an understanding of functions and equations defined parametrically and graph them
    - F-IF.10
  - Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems
    - F-IF.11

Honors Advanced Mathematics

Unit 5

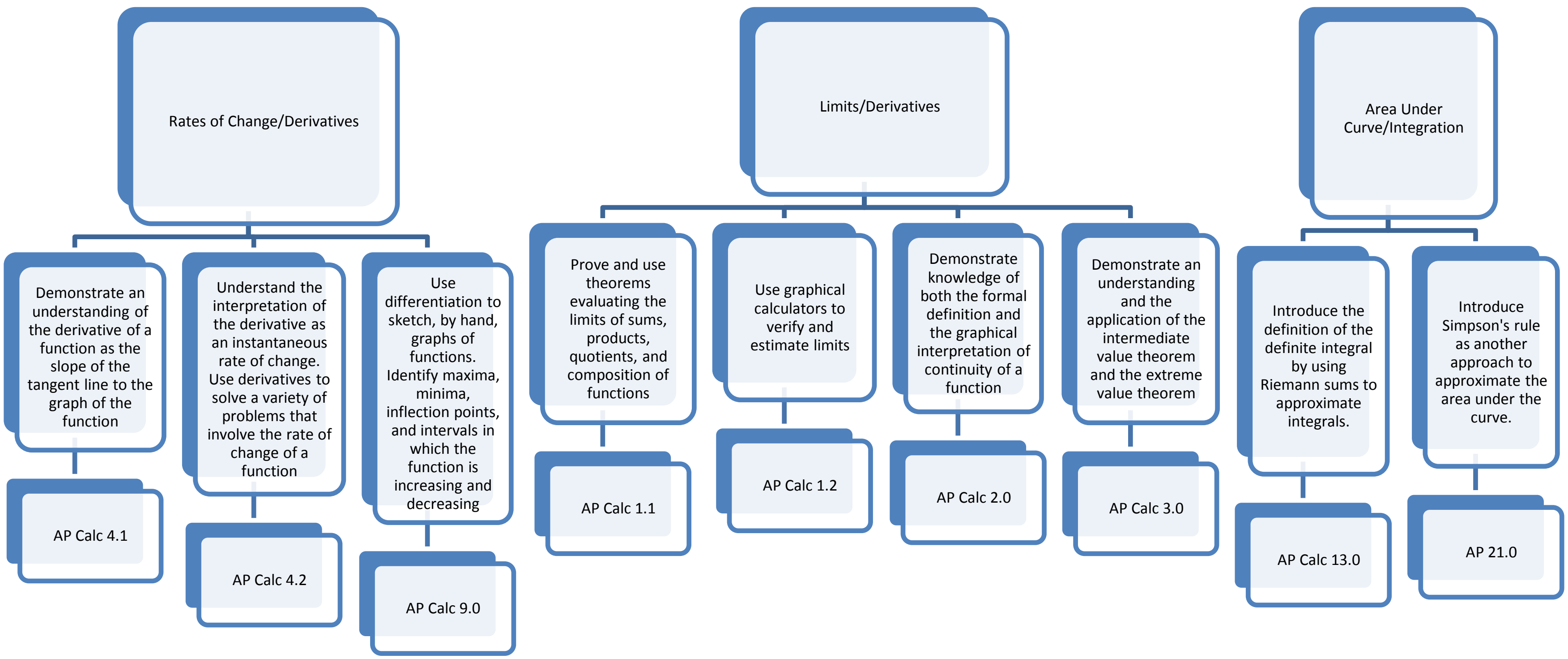
Vector and Matrix Quantities



Honors Advanced Mathematics

Unit 6

Introduction to Calculus



Honors Advanced Math – UNIT 1
Introduction & Preliminaries to Advanced Mathematics

Critical Area:
Students use reasoning to analyze equations/ inequalities and develop strategies for solving them. Through reasoning students develop fluency writing, interpreting, analyzing and translating between various forms of linear equations and inequalities. By exploring a question about the world around them (mathematical modeling) and attempting to answer the question students expand the scope of algebraic operations to solve a wide variety of linear and quadratic real world problems. Students explain why the x-coordinates of the points where the graphs y = f(x) and y = g(x) intersects and explore cases involving polynomial, rational, absolute value, exponential, and logarithmic functions.

Students connect the polynomial operations with the background knowledge of the algorithms found in multi-digit integer operations. Students realize that the operations on rational expressions (the arithmetic of rational expressions) are governed by the same rules as the arithmetic of rational numbers. Students analyze the structure in expressions and write them in equivalent forms. By modeling students expand the scope of algebraic operations to solve a wide variety of polynomial equations and real world problems. The role of factoring, as both an aid to the algebra and to the graphing of polynomials, is explored. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Create equations that describe numbers or relationships	**A-CED: Creating Equations**★ **A-CED.1.** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **CA** ★ **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ **A-CED.3.** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED.4.** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★
(m) Represent and solve equations and inequalities graphically	**A-REI: Reasoning with Equations and Inequalities** **A-REI 1.11.** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases

	where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
(m) Perform arithmetic operations on polynomials	**A-A-APR: Arithmetic with Polynomials and Rational Expressions** **A-APR 1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Understand the relationship between zeros and factors of polynomials	**A-APR 2.** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **A-APR 3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems	**A-APR 4.** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **A-APR.5** Know and apply the Binomial Theorem for the expansion of (x + y)nin powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. **(+)**
Perform arithmetic operations with complex numbers **Use complex numbers in polynomial identities and equations** *(Polynomials with real coefficients)*	**N-CN: The Complex Number System** **N-CN.1.** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real. **N-CN.2.** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.3.** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **(+)** **N-CN 7.** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.8.** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **(+)** **N-CN.9.** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **(+)**
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.**	

3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Different types of relationships between quantities can be modeled with different types of functions. • Graphs are visual representations of solution sets of equations and inequalities. • The arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. • Expressions that represent a quantity in terms of its context can be interpreted and its structure identified and rewritten. • The formula for the sum of a finite geometric series (when the common ratio is not 1) is derived and used to solve problems. • Polynomials form a system analogous to the integers which are closed under the operations of addition, subtraction, and multiplication and polynomial identities are proven to describe numerical relationships. • The Remainder Theorem can be applied for a polynomial p(x).	1. What relationships between quantities can be modeled by functions? 2. What does it mean to solve equations graphically? 3. What are the similarities and differences between linear, quadratic, and exponential functions? 4. What do extraneous solutions represent? 5. How does the arithmetic of rational numbers relate to simplifying rational expressions? 6. What does the graph of a function represent? 7. How can you represent the zeroes of a function? 8. How can you describe and show the ways you can find the zeroes (roots) of a function? 9. How can the formula for the sum of a finite geometric series be derived and used to solve problems? 10. How can you use the Binomial Theorem to expand powers of expressions? 11. What are the differences and similarities between real and complex solutions of polynomial equations? Explain graphically or algebraically.	• absolute value • binomial theorem • coefficient • complex numbers • complex roots • constraints • equations • equivalent • exponential • exponential • expressions • extraneous • factorization • factors • finite • function • functions • geometric series • inequalities

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Zeros of polynomials are identified when suitable factorizations are available and used to construct a rough graph of the function defined by the polynomial. • Binomial Theorem is for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers and known and applied. • Real and complex numbers are important in solving and understanding polynomial equations.		• infinite • interpret • linear • modeling • quadratic • quantities • radical equations • rational equations

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/ci/ma/cf/documents/aug2013algebra2.pdf **Illustrative Mathematics:** Buying a Car: A-CED.1 http://www.illustrativemathematics.org/illustrations/582 Basketball: A-CED.1 & A-REI.2 http://www.illustrativemathematics.org/illustrations/702 How Much Folate: A-CED.2 http://www.illustrativemathematics.org/illustrations/1351 Dimes and Quarters: A-CED.2 & A-CED.3 http://www.illustrativemathematics.org/illustrations/220 Growing Coffee: A-CED.3 http://www.illustrativemathematics.org/illustrations/	Most standards in the Creating Equations domain carry a modeling star, denoting their connection with the Modeling category in high school. Therefore mathematical Modeling needs to be at the forefront of conversation with students. For example, equations in high school are also more likely to contain parameters that equations in earlier grades, and so interpreting a solution to an equation might involve more than consideration of a numerical value, but consideration of how the solution behaves as the parameters are varied. Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions. Give students examples of real-world problems that can be solved by writing an equation, and have	**Smarter Balanced Assessment Consortium (SBAC)** http://www.smarterbalanced.org/ **Partnership for Assessment of Readiness for Colleges and Careers (PARCC)** Seeing Structure in an Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-equation Seeing Structure in a Quadratic Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-quadratic-equation Graph of Functions http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf Brett's Race http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

611 Bernardo and Sylvia Play a Game: A-CED.3 http://www.illustrativemathematics.org/illustrations/1010 Clea on an Escalator: A-CED.2 http://www.illustrativemathematics.org/illustrations/1003 Equations and Formulas: A-CED.4 http://www.illustrativemathematics.org/illustrations/393 Radical Equations: A-REI.2 http://www.illustrativemathematics.org/illustrations/391 Introduction to Polynomials - College Fund: A-REI.11 http://www.illustrativemathematics.org/illustrations/155	students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context. Provide visual examples of radical and rational equations with technology so that students can see the solution as the intersection of two functions and further understand how extraneous solutions do not fit the model. Have students use technology to graph and explore functions. Discuss the meaning of parameters in the graph including the table, the curves, and the solution to the equation. Have students investigate real-world examples of two-dimensional inequalities. An instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	

LANGUAGE GOALS

Writing:

1) Students will explain and justify the process of solving equations and inequalities by using key vocabulary terms.
Example: I solved for the variable in the inequality by _________. This means that _________.

2) Compare and contrast the differences and similarities between linear, quadratic, and exponential functions.
Example: The intercepts for linear graphs can be found by _________. The intercepts for quadratic functions can be found by _________. The intercepts for exponential functions can be found by _________.The intercept for the three types of graphs, are different because____________.

3) Students will explain how they developed their mathematical models.
   Example: The variables in the equation represent __________. The terms and coefficients in the equations are added/ subtracted/multiplied/ divided because __________.

Listening and Speaking:

1) Students will generate class discussions using key vocabulary terms related to solving linear, quadratic, and exponential functions.
2) Students work in pairs to explain and justify how to solve an equation and summarize their partner's explanation using various tools, such as: media, poster, graphic organizer, etc.

Reading:

1) Students will identify mathematically relevant information from real-world scenarios and model equations with them.

PERFORMANCE TASKS

Mathematics Assessment Project

- Solving Linear Equations in Two Variables: A-CED.2
  http://map.mathshell.org/materials/lessons.php?taskid=209#task209
- Optimization Problems: Boomerangs: A-CED 2
  http://map.mathshell.org/materials/download.php?fileid=1241

Illustrative Mathematics

- Combined Fuel Efficiency: A-APR.6
  https://www.illustrativemathematics.org/illustrations/825
- Population and Food Supply: A-REI 11
  https://www.illustrativemathematics.org/illustrations/645

NCTM Illuminations

- Trout Pond Population: A-CED.2
  http://illuminations.nctm.org/Lesson.aspx?id=1549
- Exploring Linear Data: A-CED.2
  http://illuminations.nctm.org/Lesson.aspx?id=1189

DIFFERENTIATION📖

FRONT LOADING	ACCELERATION	INTERVENTION

• Involve students to have a discussion that center around extending their knowledge of creating and analyzing linear equations and inequalities. Have them use their prior knowledge of graphing linear equations and inequalities to solve real world scenarios. • Engage students in an activity that would involve comparing linear equations with quadratic equations, and then quadratic equations with exponential equations. • Have students match linear, quadratic, exponential functions with their graphs, tables, and equations.	• Provide examples of real-world problems that can be modeled using linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Have students use technology to graph the functions and make tables of values. • Ask students to discover, model, and explain real-world scenarios in their everyday life that can be modeled using linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• Show students how to create numerical equations and then introduce linear equations in one variable. Students can make comparisons using numerical and linear equations. • Have students use technology to graph and generate tables of values for different types of equations. Lead student discussions about the graphs and tables of values to teach and reinforce key vocabulary terms such as intercepts, slopes, intersection, linear, roots, parabolas, etc…

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Honors Advanced Mathematics – UNIT 2
Functions, Logarithms and Trigonometry

Critical Area:

Students will develop the general understanding of functions in terms of their behavior and the properties including increasing and decreasing functions, concavity, even / odd functions, end behavior and asymptotes . They synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They graph shapes and relate the graphs to the behavior of the functions with the transformation on the variable (e.g. the graph of $y = f\ (x + 2)$).

Students expand their understanding of the trigonometric functions first developed in Geometry to explore the graphs of trigonometric functions with attention to the connection between the unit circle representation of the trigonometric functions and their properties, use trigonometric functions to model periodic phenomena.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Analyze functions using different representations	**F-IF: Interpreting Functions** **F-IF.7**. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise - defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)
(m) Building a function that models a relationship between two quantities	**F-BF: Building Functions** **F-BF.1** Write a function that describes a relationship between two quantities. ★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

(s) Building functions from existing functions	**F-BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.4** Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$
(s/a)Construct and compare linear, quadratic, and exponential models and solve problems	**F-LE: Linear, Quadratic, and Exponential Models**★ **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. [Logarithms as solutions for exponentials.] 4.1 Prove simple laws of logarithms. CA ★ 4.2 Use the definition of logarithms to translate between logarithms in any base. CA ★ 4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA ★
(s/a)Extend the domain of trigonometric functions using the unit circle.	**F-TF: Trigonometric Functions** **F-TF.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF. 2.1** Graph all 6 basic trigonometric functions. CA ★
(s/a)Model periodic phenomena with trigonometric functions.	**F-TF. 5**. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. CA ★
MATHEMATICAL PRACTICES	PROGRESSION
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the arguments of others. 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.**	http://opi.mt.gov/PDF/CCSSO/MCCS-MATH/STAGE1/Resources/2012_12-04Draft-High-School-Progression-Functions.pdf http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_modeling_2013_07_04.pdf

7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

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(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
Functions: • Different types of relationships between quantities can be modeled with different types of functions. • Functions and relations can be represented using polar coordinates. • Functions and equations can be defined parametrically. • All functions have algebraic, numerical, graphical and verbal representations. • Operations and transformations apply to all types of functions and can be used to build new functions from existing functions. • The inverse functions interchange the domain and the range. • The domain of a non-invertible function needs to be restricted in order to construct its inverse function. • Graphs of functions can explain the observed local and global behavior of a function. • Asymptotes represent the restricted domain or range. • The graph of a function demonstrates the end behavior as it approaches the vertical, horizontal or oblique asymptotes. • Real world situations can be modeled and solved by using various functions. **Logarithms:** • Logarithms are exponents. • Logarithms are used to solve exponential	1. What relationships exist between quantities that can be modeled by functions? 2. How can functions and relations be represented using polar coordinates? 3. Why is it important to define functions and equations parametrically? 4. What does it mean to solve equations graphically? 5. What do the domain and the range of a function represent? 6. What do asymptotes represent? 7. What do the maximum and minimum represent and how do they relate to the end behavior of a function? 8. How do we build new functions from existing functions using transformations? 9. What are the similarities and differences between linear, quadratic, exponential, logarithmic and polynomial functions? 10. How do we compare/contrast exponential and logarithmic functions? 11. What are inverse functions and what are they being used for? 12. How do we restrict the domain of a non-invertible function to produce an invertible function? 13. How can we use logarithms to solve an equation when the exponent is a variable? How can we change the base of a logarithm? 14. What is the angle of rotation, and how is it measured? 15. Why do we need radian measure? 16. How can sine, cosine, and tangent functions be defined using the unit circle?	• Asymptotes - horizontal, vertical and oblique • Complex roots • Composite function • Compress/ stretch • Domain/ Range • End behavior • Even/ odd functions • Exponential • Frequency • Increasing/decreasing • Intercepts • Inverse function • Invertible, non-invertible • Laws of Logarithms • Logarithmic expressions • Logarithmic functions • Maximum/ minimum • Midline • One-to-one functions • Period, amplitude, phase shift • Periodicity • Piecewise function • Quadrantal and coterminal angles

equations. • The definition of logarithms can be used to translate between logarithms in any base. **Trigonometric functions:** • Trigonometric relationships and functions can be used to model real-world phenomenon. • Indirect measurements of lengths and angles can be used to solve a variety of problems. • Domain must be limited to finding the inverse of a trigonometric function. • Inverse functions must be used to find solutions in some modeling problems. • A circle is a set of points that can be defined by an equation. This measurement is expressed in radians rather than degrees. • Students extend the domain of trigonometric functions using the unit circle. • Students establish a way to measure angles with respect to arc length. • The trigonometric functions are extended to all real numbers to describe rotations around the unit circle. • Sine, cosine, and tangent functions can be defined using the unit circle. • Our world is periodic. The amount of sunlight a city receives on a given day, high and low tides are all real life instances where sinusoids explain and model real life phenomena.	17. What are periodic functions and why is modeling them so important? 18. Why is the Theorem of Pythagoras so essential in trigonometry? 19. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. 20. How can the graphs of the sine, cosine, tangent functions and their inverses be compared? 21. How can you use the addition and subtraction formulas for sine, cosine, and tangent to solve problems? 22. What do we do to find the inverses of trigonometric functions? 23. How can you solve trigonometric equations using the inverse functions? 24. What are the period, amplitude, and midline of the graph of a trigonometric function? 25. How can technology be used to evaluate solutions of trigonometric functions?	• Radian Measure • Rational functions • Reflection over the x and y-axis • Relative Minimum • Restricted domain • Sine, cosine, tangent • Sinusoidal graphs • Secant, cosecant, tangent • Step function • Symmetries • Transformations • Trigonometric functions • Vertical/ horizontal shifts

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Illustrative Mathematics • Functions: https://www.illustrativemathematics.org/standards/hs • Graphic Representations of the Real Life Situations http://graphingstories.com/ • Applications of Trigonometry: www.math12.com • Prentice Hall Algebra Two Online resources: http://www.phschool.com/atschool/phmath07/program_page_hs.html **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum (2007)	**Unit 2 is largest unit** which includes the variety of major topics such as interpreting and building functions including the logarithmic and trigonometric Functions. Students will be analyzing functions, graphing with transformations, comparing/contrasting functions graphs and solve real-world problems. Students are required to understand families of functions and the inverse of those functions. Students must be familiar with the concept and formal definition of inverse functions, namely that if $f \circ g(x) = g \circ f(x) = x$, then $f(x)$ and $g(x)$ are inverses of one another. Teachers should first work with evaluating functions, then composing general functions and finally composing inverse functions. Once students have mastered the composition of inverse functions, they should be made to derive the inverse functions and prove that they have found the inverse by using the above definition. Students should recall parent functions $f(x)$ and then explore the effect of $f(x)+k$, $f(x+k)$, $kf(x)$, $f(kx)$ on the graph for all k. The mathematical progressions demand that students are fluent with the parent functions and can use them quickly to determine the graph of transformed functions. Students will explore the relationship between functions and their inverses on the same coordinate plane. They will use that understanding to then explain the connection between the line of symmetry of the two functions and the algebraic method of letting $f(x) = x$ and $x = f^{-1}(x)$ to solve for the inverse function $f^{-1}(x)$. Students should then come to understand why a function needs to be one-to-one in order to have an inverse and then why it is necessary and possible to restrict a domain on a function to create an invertible function. Provide visual examples of transformed functions while manipulating different constants in the function parameters. Have students use technology to manipulate the parameters of the functions and record how the parameters affect the graphs and tables of the functions.	SBAC – http://www.smarterbalanced.org/ PARCC - http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

	Use a compass and straightedge to explore a unit circle with a fixed radius of 1. Help students to recognize that the circumference of the circle is 2π, which represents the number of radians for one complete revolution around the circle. Students can determine that, for example, $\pi/4$ radians would represent a revolution of 1/8 of the circle or 45°. Students can examine how a counterclockwise rotation determines a coordinate of a particular point in the unit circle from which sine, cosine, and tangent can be determined. Have students explore real-world examples of periodic functions; such as: average high (or low) temperatures throughout the year, the height of ocean tides as they advance and recede, and the fractional part of the moon that one can see on each day of the month. Graphing some real-world examples can allow students to express the amplitude, frequency, and midline of each. • *Teachers might find the following strategies helpful:* **Cooperative learning**: Engage all students by using a variety of differentiation strategies including but not limited to questioning techniques, wait time, Think-Pair-Share, peer tutoring, small groups collaboration Students are required to understand families of functions and the inverse of those functions. **Checking for understanding and reflecting on students' background knowledge:** Use a variety of strategies to frequently check for understanding such as small white boards, hand signals (thumbs up/thumbs down), parking lot questions, etc. Teachers make connections to students' prior knowledge. **Problem-solving and abstract reasoning:** Analyze the data, compare/contrast, use counterexamples, construct plausible arguments, make conclusions, justify different ways to solve a problem and communicate to others. **Quick write:** Explain the process and the solution by using academic language and key vocabulary. **Modeling and solving real-world problems:** Apply algebraic skills and knowledge to solve a variety of engaging/ relevant problems to make assumptions, analyze the data, derive to solutions and draw viable conclusions. **Technology-enhanced instruction:** Utilize graphing calculators, spreadsheets, computer algebra systems, statistical packages and other appropriate software. **Project-based learning:** Use a variety of problem-solving	

	assignments such as creating/ solving word problems and the culminating unit tasks.	

LANGUAGE GOALS

Writing:

1) Students will explain and justify in writing the behavior of the function as it approaches horizontal and vertical asymptotes.
Example: As the function approaches positive infinity along the x-axis, the graph of the function approaches the horizontal asymptote from above.
2) Students will explain (in writing and orally) the effects of transformations on a function and test that understanding for all parent functions.
Example: The transformation $f\left(x+a\right)+b$, *moves the parent function* $-a$ *units in the horizontal direction and* b *units in the vertical direction.*
3) Students will compare and contrast (in writing and orally) the differences and similarities between linear, polynomial, and exponential functions.
Example: All three functions increase as x increases. Polynomial and exponential functions are curves and the linear function is a line. Exponential functions will increase at a faster rate than polynomial functions.
4) Students will write about the relationship between the inverse of functions and the concept of rotating the axes about the line of symmetry to determine the inverse function.
Example: The inverse function can be determined by rotating the function of the graph about the line of symmetry. This is algebraically equivalent to interchanging the x and y values in a function and solving for y.
5) Students will write about how functions can be used to in real life to facilitate repeated algorithms.
Example: Computers often make use of functions to run programs i.e. clicking on the icon for Internet Explorer will run a function to launch a program that connects the modem to the internet and opens a screen to a preselected page.
6) Students will explain in writing how to prove and apply the Laws of Sines and Cosines using technical vocabulary in complex sentences.
7) Students will explain (in writing and orally) the terms and definitions of the trigonometric functions; conic sections; and complex numbers.
Example: To find the (<u>amplitude</u>) of the function, I can first find the (<u>midline</u>) and then find the distance to the (<u>maximum or minimum</u>) of the graph.

Listening and Speaking:

1) Students will participate in class discussions using specific vocabulary related to transformations and functions.
2) Students will explain and justify (orally) how to graph a function to a partner as well as restating and summarizing their partner's explanation.
Example: First I __________ because ____________, second I ___________ because__________,…

Reading:

1) Students will identify the relevant information and details in a passage and create a single function that represents a composition out of many subparts.

PERFORMANCE TASKS

<u>Precalculus Enhanced with Graphing Utilities</u>, Sullivan & Sullivan, 4th Edition (2005), **ISBN-10:** 0131490923

F-IF.7d

- Population Model, Page 197, # 53 and 54
- Cost of a Can, Page 210, # 61
- **Waves, Chapter Project**, Page 515, # 1
- Discussion and Writing, Page 270, # 84-90

<u>Precalculus Graphical, Numerical, Algebraic</u>, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education 2007

F-IF.7d

- Designing a Cardboard Box, Page 265, # 59
- Industrial Design, Page 272, # 94 and 95
- Designing a Juice Can, Page 265, # 61

Illustrative Mathematics:

1) F-IF.9 Throwing Baseballs : https://www.illustrativemathematics.org/illustrations/1279
2) F-BF.1 Compounding with a 5% Interest Rate: https://www.illustrativemathematics.org/illustrations/572
3) F-BF.3 Transforming the graph of a function : http://www.illustrativemathematics.org/illustrations/742
4) F-BF.3 Building an Explicit Quadratic Function by Composition: www.illustrativemathematics.org/illustrations/744
5) F-LF.4 Carbon 14 Dating : https://www.illustrativemathematics.org/illustrations/369
6) F-TF. 1 Bicycle Wheel: https://www.illustrativemathematics.org/illustrations/1873
7) F-TF.5 As the Wheel Turns: https://www.illustrativemathematics.org/illustrations/595
8) F-TF. 5 Foxes and Rabbits 2 : https://www.illustrativemathematics.org/illustrations/816
9) F-TF.5 Foxes and Rabbits 3: https://www.illustrativemathematics.org/illustrations/817
10) F-TF.5 Exploring Sinusoidal Functions F-TF.5 https://www.illustrativemathematics.org/illustrations/1647

DIFFERENTIATION

FRONT LOADING	ACCELERATION	INTERVENTION
• Have students recall how to graph by hand linear, quadratic and cubic functions from a table of values and then understand how to graph all parent functions. • Get the students to explain how to solve quadratic equations by the quadratic formula and completing the square. • Engage students in an activity that would involve comparing linear functions with quadratics functions, and then quadratics functions and exponential functions. • Involve students in the processes required to solve equations and start to discuss the concept of inverse functions. • Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations.	• Students work in small groups with a curriculum that is conceptually demanding as well as rigorous due to the speed at which the course moves and the concepts covered. Students collaborate and concentrate on tasks for extended periods of time, to contribute to discussions, to predict and test their predictions. • The assessments for advanced students will demand the ability to apply learned concepts to solving abstract or real world problems or summarize the patterns/ concepts learned. Students will use the "Socratic Method" for posing questions to discover connections, patterns and structure. • Students learn about the modeling of real world data with polynomial functions, rational functions, exponential functions, radical functions, logarithmic functions, and sinusoidal functions. They explore in depth the various characteristics of functions, i.e.	• Reflect on students prior knowledge of the following Algebra 1 topics: • Radicals and exponents, rational expressions and equations, operations with polynomials and the basic graphic techniques. • Review the difference between independent events and dependent variables. • Review the difference between real and complex roots and the operations with complex numbers. • Review how to create tables of values and to use those values to generate the graph of the function. • Review key vocabulary words from unit 1. • Allow students to use technology to

• Involve students in the discussion on zeros of polynomial functions and their roots/zeros. • Have students recall the properties of exponents including rational exponents. • Check students understanding of the geometric transformations such as translations and reflections. • Have students recall the trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	rates of change, concavity, inverses, continuity, discontinuity and asymptotes. Students further explore functions in terms of composite and inverse functions, their transformations and periodicity. • Students work on projects to apply these concepts to real-world problems by creating equations and exploring the graphs of those equations using technology application to determine which parts of the graph are relevant to the problem context.	quickly generate a table of values after they have shown some skill in evaluating expressions by hand. • Using technology, students work in small groups to graph different functions and compare/contrast the graphs and make conclusions.

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Honors Advanced Mathematics – UNIT 3
Statistics and Probability

Critical Area:

Students analyze data to make sound statistical decisions based on probability models. By investigating examples of simulations of experiments and observing outcomes of the data, students gain an understanding of what it means for a model to fit a particular data set. Students develop a statistical question in the form of a hypothesis (supposition) about a population parameter, choose a probability model for collecting data relevant to that parameter, collect data, and compare the results seen in the data with what is expected under the hypothesis. Students build on their understanding of data distributions to help see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). In addition, they can learn through examples the empirical rule, that for a normally distributed data set, 68% of the data lies within one standard deviation of the mean, and that 95% are within two standard deviations of the mean.

CLUSTERS	COMMON CORE STATE STANDARDS
(s)Summarize, represent, and interpret data on a single count or measurement data.	**Statistics and Probability – Interpreting Categorical and Quantitative Data** **S.ID.4.** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Understand and evaluate random processes underlying statistical experiments. **Make inferences and justify conclusions from sample surveys experiments, and observational studies.**	**Statistics and Probability – Making Inferences and Justifying Conclusions** **S.IC.1.** Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. **S.IC.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **S.IC.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S.IC.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S.IC.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **S.IC.6.** Evaluate reports based on data**.**
Use probability to evaluate outcomes of decisions.	**Statistics and Probability – Using Probability to Make Decisions** **S.MD.6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7.** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

LEARNING PROGRESSIONS
Draft High School Progression on Statistics and Probability http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• In real life, data sets are large and almost always approximately normal. Normal models which include estimation of areas under the normal curve allow us to answer and model real life situations. • Sampling methods, when highly representative of a population, allow accurate predictions or inferences of population parameters. • Students model probabilities found in experimental environment and decide whether they are consistent with theoretical probabilities? • The mean or proportion of a sample is the same as the mean or proportion of a population, within a margin of error. • If the difference between the statistics of two	• Why do we study normal distributions? • Why is random sampling of a population done when a census is impractical? • Do experimental probabilities match theoretical probabilities? • How can a researcher select a method of collecting data with as little bias as possible? • How does the mean or proportion of a sample compare to the mean or proportion of the population? • When does a statistic become extraordinary instead of ordinary? • How do you know when the difference between two treatments is statistically significant.	Bell curve bias categorical data census complementary events conditional probability confidence interval convenience sample correlation coefficient counting methods critical value of z distribution experimental probability experimental study

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
treatments is outside of a critical confidence interval, the difference is statistically significant. • Select a method of gathering data from a random sample and understand data by critically differentiating the merit of reports and data encountered in daily life. • Probability can be used to develop strategies and make informed decisions.	• There are many "studies out there", how do I know if they are really accurate? • How can probability be used to make fair decisions?	fairness Histogram independence independent events margin of error mean (x-bar) normal model or normal distribution null hypothesis Numerical data observational study parameter population probability distribution proportion (p-hat) qualitative data random number generator random sample random variable representative sample sampling significant (as in statistics) simple random sample standard deviation statistic stratified random sample Subject survey systematic random sample theoretical probability treatment voluntary sample Z-Score

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **Illustrative Mathematics:** School Advisory Panel: S-IC.1 http://www.illustrativemathematics.org/illustrations/186 Strict Parents: S-IC.1, 3 http://www.illustrativemathematics.org/illustrations/122 Musical Preferences: S-IC.1, S-ID.5 http://www.illustrativemathematics.org/illustrations/123 SAT Score: S.ID.4 http://www.illustrativemathematics.org/illustrations/216 Do You Fit In This Car?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1020 Should We Send Out a Certificate?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218	Engage students in a discussion or activity to clearly distinguish between categorical and numerical variables by providing multiple examples of each type. Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's sport preferences) or measurement on the vertical (students' grade in a test). One tool for developing statistical models is the use of simulations. This allows the students to visualize the model and apply their understanding of the statistical process. Provide students the opportunities to distinguish between a population parameter which is a constant, and a sample statistic which is a variable. Use teacher-guided comparison conversations to ensure that students are able to make connections. As the statistical process is being mastered by students, it is important for them to investigate questions such as "If a coin spun five times produces five tails in a row, could one conclude that the coin is biased toward tails?" Students will need to use all of the data analysis, statistics, and probability concepts covered to date to develop a deeper understanding of inferential reasoning. Have students critique published surveys before having them design their own surveys. Unlike	SBAC - http://www.smarterbalanced.org/

	in observational studies; in surveys, the sample selected from a population needs to be representative of the population. Taking a random sample is generally what is done to satisfy this requirement. Use a variety of devices as appropriate to carry out simulations: number cubes, cards, random digit tables, graphing calculators, computer programs.	

LANGUAGE GOALS

- Students will describe orally and in writing the purposes of and differences among sample surveys, experiments, and observational studies.
  Example: "Based on the survey of teenage high school students, more students are more/less likely to _____ than _____."

- Students will decide whether a specified model is consistent with results from a data simulation.
  Example: "A model stating that a spinning coin falls heads up with a probability of 0.5 is not consistent with a simulation result of 5 tails in a row."

- Students will explain orally and in writing how they use statistical and probability concept in their lives, using the following specific set of words: *distribution*, *mean*, *standard deviation*, *probability*, and *statistics.*
  Example: "Based on the distribution of test scores with a mean of _____ and a standard deviation of _____, a test score of _____ is (*adjective).*
- Students will explain orally and in writing that areas under the normal curve allow us to answer and model real life situations.

PERFORMANCE TASKS

Mathematics Assessment Projects (MARS Tasks)

- **Modeling Conditional Probabilities 1: Lucky Dip: S.MD.6** http://map.mathshell.org/materials/lessons.php?taskid=409&subpage=problem

NCTM Illuminations Lessons

- **Should We Send a Certificate?**: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218
- **Exploration with Chance:** S.ID.6 http://illuminations.nctm.org/LessonDetail.aspx?id=L290

Illuminations
Fred's Fun Factory: S-MD.2, 5 and 7
http://www.illustrativemathematics.org/illustrations/1197
Miscellaneous Sources

- **The Normal Distribution: S.ID.4** http://www.wmich.edu/cpmp/1st/unitsamples/pdfs/C3U5_362-375.pdf
- **Applications of Probability:** http://www.schools.utah.gov/CURR/mathsec/Core/Secondary-II/II-4-S-MD-H-6-and-7.aspx

DIFFERENTIATION📖		
FRONT LOADING	ACCELERATION	INTERVENTION
Students should be encouraged to persevere when problem solving in this unit. Multiple solutions are common and should be recognized. Students can often make sense of complex contextual probabilities by considering a simpler analogous Probability situation (MP.1). As students work to identify events for which probabilities are to be determined and rules to apply, encourage students to verify and critique the thinking of their classmates (MP.3). Students have the opportunity to demonstrate proficiency with MP.6 by paying close attention to precise use of new vocabulary and writing complete sentences describing probabilities.	S.MD.7 Apply this standard with more complex probability models. You can implement the following activity: But mango is my favorite… http://www.illustrativemathematics.org/illustrations/1333 Often two sample groups are compared in clinical studies. Two key criteria are specified: are the data normally distributed and are the data paired? Unpaired (independent) normally distributed data: Student's unpaired two-sample t-test For example, the efficacy of a new drug A may be compared with an established drug B. The study has 220 patients in treatment Group A with sample mean $\bar{x}A$ and standard deviation SDA and 200 patients in treatment Group B with sample mean $\bar{x}B$ and standard deviation SDB; (Group A and Group B do not have to be equal). We need to calculate the difference between the two sample means and the standard error of this difference between the two means, from which we can calculate a confidence interval for the difference between them. For t-test to be valid, the standard deviations of both groups must be similar. This is often the case, even when the sample means are significantly different.	Review the difference between independent events and dependent events. Review the conversions of: • Ratios • Percentages • Decimals Teach students how to understand data in multiple forms: • Graphs • Charts • Table Review key vocabulary words from previous sections

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Honors Advanced Math – UNIT 4
Polar Equations and the Complex Plane

Critical Area:
Students derive the equations of ellipses and hyperbolas given foci. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, they use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola as well as graph the equation.

Students investigate the geometry of the complex numbers more fully and connect it to operations with complex numbers. They represent complex numbers on the complex plane in both rectangular and polar form. They calculate the distance between numbers in the complex plane as the modulus of the difference. Students expand the skills involved in working with equations into several areas: trigonometric functions, by setting up and solving equations such as $sin\, 2\theta = ½$; parametric functions by making sense of the equations $x = 2t, y = 3t + 1, 0 \leq t \leq 10$. Students develop conceptual knowledge of functions that set the stage for the learning of other standards in Precalculus. They investigate the relationship between the graphs of sine and cosine as a function of theta and also use the parametric form of the functions where $x(\theta) = cos(\theta)$ and $y(\theta) = sin(\theta)$.

CLUSTERS	COMMON CORE STATE STANDARDS
Translate between the geometric and the equation for a conic section	**Geometry: Expressing Geometric Properties with Equations** **G-GPE.3. (+)** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is consistent. **G-GPE.3.1**. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola, and graph the equation
Represent complex numbers and their operations on the complex plan	**N-CN.4.(+)** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN. 5. (+)** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has *modulus 2 and argument 120°*. **N-CN.6**. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

(m) Interpret functions that arise in applications in terms of the context	**F-IF.10. (+)** Demonstrate an understanding of functions and equations defined parametrically and graph them. CA **F-IF.11. (+)** Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. CA
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning**.	

LEARNING PROGRESSIONS

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The sum or difference of the distances of the foci from the directrix is consistent. • Graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$ can be circles, parabolas, ellipses, or hyperbolas. • The equations of ellipses and hyperbolas can be derived from the foci. • The relationship between the graph of a complex number and their operations and the conjugation of complex numbers on the	1) What are the geometric characteristics of conics? 2) How do you identify the graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$? 3) How can you graph a complex number in rectangular and polar form? 4) What is the relationship between rectangular and polar form of a complex number? 5) What is the importance of knowing the conjugate of a complex number? 6) In terms of their respective equations, what is the difference between a circle and an ellipse?	circle directrix eccentricity ellipses foci hyperbolas parabola parametric function complex number complex plane real axis

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
complex plane can be understood. • Different types of relationships between quantities can be modeled with different types of functions. • Functions and relations can be represented using polar coordinates. • Functions and equations can be defined parametrically. • All functions have algebraic, numerical, graphical and verbal representations. • Graphs are visual representations of solution sets of equations and inequalities. • Graphs of functions can explain the observed local and global behavior of a function. • Real world situations can be modeled and solved by using various functions	7) What relationships exist between quantities that can be modeled by functions? 8) How can functions and relations be represented using polar coordinates? 9) Why is it important to define functions and equations parametrically? 10) What does it mean to solve equations graphically? 11) What do the domain and the range of a function represent?	imaginary axis magnitude modulus argument conjugate polar form polar coordinates rectangular form parametric equations parametric curves

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
NCTM Illuminations •*Cutting Conics*:G-GPE.3 Students explore and discover conic sections by cutting a cone with a plane. Circles, ellipses, parabolas, and hyperbolas are examined using the Conic Section Explorer tool. Physical manipulatives such as dough can optionally be used as well. http://illuminations.nctm.org/Lesson.aspx?id=2907 *Human Conics:* G-GPE.3 Students use sidewalk chalk and rope to illustrate the locus definitions of ellipses and parabolas. Kinesthetics, teamwork, and problem solving are stressed as students take on the role	Students will explore the conic sections and describe how to cut a cone to create the various conic sections. Separate the class into 6 groups (or a multiple of 6 if your class is large). Assign two conic sections to each group. There are 6 different ways to do this: circle/ellipse, circle/hyperbola, circle/parabola, ellipse/hyperbola, ellipse/parabola, and hyperbola/parabola. Each group should create a poster summarizing what they've learned about their two conic sections and comparing and contrasting them. Students will write a summary of either the ellipse or parabola construction for the benefit of a classmate who has missed the lesson. The	1. Ask students to describe how they discovered how to cut their cones to create each conic section–circles, ellipses, parábolas, hyperbolas. 2. Give students a picture of an ellipse and a parabola with possible foci or directrix indicated. Ask them to use a ruler and right angle measure to determine and explain whether or not the figure is actually the named conic. 3. Using data regarding the distance from the Sun and the orbital periods of other planets, ask students to generate parametric equations for the orbits of the other planets in

of focus, directrix, and point on the conic, and figure out how to construct the shape. http://illuminations.nctm.org/Lesson.aspx?id=3003 *Mars Orbit:* F-IF.10 Students will generate parametric equations to describe the position of planets relative to the Sun; then, they will combine the equations to describe the position of Mars relative to Earth.http://illuminations.nctm.org/Lesson.aspx?id=3980 *Axonometry*: N-CN.4, N-CN.5 http://illuminations.nctm.org/Lesson.aspx?id=4228 Wolfram Demonstration: N-CN.4 http://demonstrations.wolfram.com/ComplexNumber/ Engage New York https://www.engageny.org/sites/default/files/resource/attachments/precalculus-m1-module-overview-and-assessments.pdf CN.4 and N-CN.5 **LAUSD Adopted Textbooks** <u>Precalculus Enhanced with Graphing Utilities</u>, 4th Edition, Sullivan & Sullivan, Pearson/Prentice Hall (2005). <u>Precalculus Graphical, Numerical, Algebraic</u>, 7th edition, Demana, Waits, Foley & Kennedy, Addison	summary should include the definition and an explanation of how the drawing technique applies the definition. Afterwards, students can exchange and critique their summary with other students. Given parametric equations, group students and ask them to find the polar equation that will give the same shape as the one obtained with given parametric equations. Afterwards, students will share their explanations in a whole class discussion. Use properties of difference of two squares to find the modulus. Relate the modulus visually using vectors. Graph complex numbers and identify the magnitude of the complex number, the distance of the complex number from the origin, and the direction of the complex number from the origin. Express complex numbers in polar coordinate form and in rectangular form. Have students explore the conic sections and describe how to cut a cone to create various conic sections. Import images of circles from fields from Google Earth into a coordinate grid system and find their equations	the solar system relative to the Earth

Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2ndedition, Foerster, Key Curriculum (2007)		

LANGUAGE GOALS

Writing:

1) Students will explain and justify the process of completing the square to identify whether the quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$ is a ellipse, circle, parabola, or a hyperbola.

Example: I completed the process of completing the square by _____ and found that _____. This means that graph of the quadratic equation is a _______.

2) Students will compare and contrast the differences and similarities between ellipses, circles, parabolas, and hyperbolas.

Example: If the eccentricity of a conic section is _______, than the graph is a _______.

Listening and Speaking:

3. Students will generate class discussions using academic vocabulary related to the rectangular and polar forms of complex numbers.

4) Students will participate in class discussions using specific vocabulary related to transformations and functions.

5) Students will explain and justify(orally) how to graph a function to a partner as well as restating and summarizing their partner's explanation.

Example: First I __________ because ____________, second I ___________ because__________,

Reading:

6. Students will read a word problem and identify the language needed to create an algebraic representation in order to solve the problem.

PERFORMANCE TASKS

Textbook:

Larson, R. and Hostetler, R. (2007). Pre-Calculus with Limits, 5th edition. Boston, New York: Houghton/Mifflin.

Publisher: Houghton Mifflin Company

Authors: Larson, R., Hostetler, R.

Topic: Conics

- Suspension Bridge (Page 742, Problem 62)
- Loran (Page 761, Problem 42)
- Satellite Tracking (Page 798, Problem 58)
- Earthquake: Page 667 #3
- Road Design: Page 669 #94
- Architecture: G-GPE.3Page 678 #47-49
- Navigation: Page 688 #46
- Planetary Motion: Page 727 #55

DIFFERENTIATION 🕮		
FRONT LOADING	ACCELERATION	INTERVENTION
Introduce students to ellipses and help them understand that conics are like circle and parabolas. Introduce students to the equations and graphs of conics and help them see the relationship between equation and graph. Engage students in an activity that would connect their understanding of conics to the real-world. Illustrate conic sections geometrically as cross-sections of a cone. Have students define conic sections and illustrate it pictorially.	Provide examples of real-world problems that can be modeled by circles, parabolas, and ellipses. Students will write and graph equations in polar form. Students will classify conics from their general equation. Students will use properties of parabolas, ellipses, and hyperbolas to model and solve real-life problems	Have students use calculators or computer software to lessen the computational burden in simplifying and graphing conics. Use hands-on activities to allow students to explore how conics may vary (i.e. Using a string and two thumbtacks, have students explore how to obtain ellipses that are long or narrow) Precalculus intervention should include strategies such as targeted grouping peer and counseling grouping, Use informal techniques frequently during regular class time to gauge student understanding. Use questioning that focuses on student thinking and reasoning to help you monitor your students.

		Incorporate writing activities and group work to observe student thinking and identify misconceptions and gaps in understanding. Have students illustrate concepts using drawings, graphs, and models. Many students who need intervention struggle to learn concepts because they may not be able to grasp abstract concepts. Whenever possible, vary your instructional techniques to include use of models, manipulatives, and technology

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1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics).* Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). Progressions for the Common Core State Standards in Mathematics.
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4. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-algebra-i.
5. Mathematics Assessment Resource Service, University of Nottingham. (2007 -2012). Mathematics Assessment Project. Retrieved from
6. http://map.mathshell.org/materials/index.php..Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
7. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
8. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
9. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
10. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
11. .Larson, R. and Hostetler, R. (2007). Pre-Calculus with Limits, 5th edition. Boston, New York: Houghton/Mifflin.

12. Sullivan,M. &SullivanIII,M.(2006). Precalculus Enhanced with Graphing Utilities, 4th edition.New Jersey:Pearson, Prentice Hall.
13. Demana,F.D.,Waits, B.K., Foley, G.D. & Kennedy,D.(2007).
14. Precalculus Graphical, Numerical, Algebraic, 7th edition. Addison Wesley, Pearson Education.
15. 13.Foerster,P. A.(2007). Precalculus with Trigonometry Conceptsand Applications, 2nd edition. Emeryville, CA:Key Curriculum

Honors Advanced Math - UNIT 5
Vectors and Matrix Quantities

Critical Area: Students work with vectors, representing them geometrically and perform operations with them. They connect the notion of vectors to the complex numbers. Students also work with matrices and their operations, experiencing for the first time an algebraic system in which multiplication is not commutative. Finally, they see the connection between matrices and transformations of the plane, namely: that a vector in the plane can be multiplied by a 2x2 matrix to produce another vector, and they work with matrices from the point of view of transformations. They also find inverse matrices and use matrices to represent and solve linear systems.

CLUSTERS	COMMON CORE STATE STANDARDS
Represent and model with vector quantities	**Number and Quantity – Vector and matrix Quantities** **N-VM.1**. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, **v**). **N-VM.2**. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.3**. (+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors	**A-VM.4**. (+) Add and subtract vectors a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v – w** as **v + (-w**), where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **A-VM.5**. (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). b. Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|**v**. Compute the direction of cv knowing that when |c|**v** $\neq$ 0, the direction of c**v** is either along **v** (for c>0) or against **v** (for c<0)
Perform operations on matrices and use matrices in applications	**A-VM.6**. (+) Use matrices to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. **A-VM.7**. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

	A-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions. **A-VM.9**. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **A-VM.10**. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse. **A-VM.11**. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformation of vectors. **A-VM.12**. (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area
MATHEMATICAL PRACTICES	
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**	

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The addition of complex numbers is connected to the addition of vectors. • Matrices could be used to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. • Vectors and polar coordinates are useful in solving real-world problems. • Matrix operations could be performed on matrices and it can be an approach for solving systems of equations.	1) How are complex number addition connected to vector addition? 2) Why are functions and relations represented by vectors? 3) Why are functions represented by polar equations? 4) How are complex numbers connected to polar coordinates?	• horizontal/Vertical component • magnitude • modulus • vector quantity • scalar quantity • initial point • terminal point • position vector • scalar product • unit vector

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
		• equivalent vector • vector plane • resultant (sum)

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
• *Pick's Theorem as a System of Equations:* A-VM.6 The main problem in this lesson is to determine the values of the coefficients and the constant term in Pick's Theorem. In particular, what are the values of coefficients a and b, as well as the constant term c, in the following equation: *Area = a (Number of Perimeter Pins) + b (Number of Interior Pins) + c* http://illuminations.nctm.org/Lesson.aspx?id=2089 • *Sums of Vectors and Their Properties:* A-VM.4 This lessons illustrates how using a dynamic geometrical representation can help students develop an understanding of vectors and their properties, as described in the Number and Operations Standard. Students manipulate two vectors to control the movement of a plane in a game-like setting. Students extend their knowledge to further investigate the system of vectors. http://illuminations.nctm.org/Lesson.aspx?id=1590 • *Components of a Vector:* N-VM.1 In this lesson, students manipulate a velocity vector to control the movement of a car in a game setting. Students learn that vectors are composed of two	Students will investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Through working with vectors and matrices both geometrically and quantitatively, students discover that vector addition and operations observe their own set of rules (i.e. multiplication is not commutative, it is possible that AB = AC but B≠C, it is possible that A≠0 & B≠0 but AB = 0, etc…). Students find inverse matrices by hand in 2x2 cases and use technology in other cases. Provide examples of real-world problems that can be modeled by writing equations and solved with matrices. Begin with simple equations in two variables and build up to more complex equations in three or more variables that may be solved using matrices and technology applications. *For example*: Your school's academic club is planning the end of the year party. You have determined that the cost of admission is $13.50 for non-members and $10.35 for the academic club members, and there is a limit of 40 students. You have $500 to spend. Use an inverse matrix to determine how many members and how many non-members of the academic club to invite. Have students investigate of real-world problems that can be represented and modeled with vector quantities. Students need to decide on a solution path and make use of tools (i.e. calculators, dynamic geometry software, or spreadsheets). *For instance*: Given the speed of an aircraft and its bearing (coordinates) students would find the resultant	**Illuminations** 1. Use Gauss' theorem to see if the points A(3, 6), B(2, –3) and C(6, –2) generate a cube. Then look for a pattern in the coordinates of these points. Use the pattern to generate other numbers that also the pattern always work? 2. Ask students to write a letter to an absent algebra student providing an explanation of the technique used in class, why it worked, and some of the pitfalls that must be avoided in generating this system of equations. http://illuminations.nctm.org/Lesson.aspx?id=2089 PARCC - http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

components: magnitude and direction. http://illuminations.nctm.org/Lesson.aspx?id=1589 **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). *Pre-Calculus with Limits: A Graphing Approach*, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition, Foerster, Key Curriculum (2007)	speed and direction of the aircraft by simulating the velocity of wind effects on all four nautical directions. Facilitate whole class or small group instructional conversation throughout. Instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	

LANGUAGE GOALS

Writing:
1. Students will explain in writing how vectors as geometric objects in the plane can be represented by ordered pairs, and matrices that act on vectors.
2. Students will describe in writing an understanding of vectors and their properties.
3. Students will write equations and solve with matrices to investigate real-world problems
Example: Vector multiplication by a scalar means __________.
Speaking:
4. Students will explain (orally and in writing) the mathematical processes used in class in generating systems of equations and why it worked.
Example: The variables represent__________, and the coefficients represent ___________ because__________,…

PERFORMANCE TASKS

Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York, 2008.

Vectors in the Plane: Page 436 #91

Vectors and Dot Products: Page 446 #61

Linear Systems & Matrices: Page 484 #78

Operations with Matrices: Page 539 #82

Applications of Matrices & Determinants: Page 567-568 #27

Vector Tasks
https://docs.google.com/document/d/1lcRE17bVBhIZizwsHWEVOhREwu-PIOafvmc-hVoAxjA/edit

http://illuminations.nctm.org/unit.aspx?id=6081

http://illuminations.nctm.org/Activity.aspx?id=3536

http://illuminations.nctm.org/Lesson.aspx?id=1589

Matrices Tasks
http://illuminations.nctm.org/unit.aspx?id=6045

FRONT LOADING	ACCELERATION	INTERVENTION
Have students work with matrices and their operations in order for them to experience that matrix multiplication is not commutative.	Students will be able to apply the arithmetic of vectors and use the concept of vector to solve real-world problems. Students will be able to use matrix methods to solve and interpret systems of linear equations	Have students use calculators or computer software to lessen the computational burden in working with matrices. Vary amounts of time devoted to exploring problems. Stress the importance of using multiple representations in the examples by showing students mathematical modeling techniques.

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Honors Advanced Math – UNIT 6
Introduction to Calculus

Critical Area:
Students investigate average rate of change and focus on the numeric analysis of change over a short intervals of time, leading to discussion of limit, and instantaneous rate of change. Students engage in intuitive understanding of limiting process. They calculate limits using algebra. Given the correct notation, students practice and apply the definition of limit with various kinds of functions, including piece-wise defined functions with a step-discontinuity. Students formally define limit, and practice existence proofs of limits as x approaches a fixed number. Students justify answers analytically, graphically, numerically, and verbally and construct viable argument regarding the non-routine problems posed. Students understand the interplay between the geometric and analytic information and use calculus to predict and to explain the observed local and global behavior of a function. Students numerically analyze curves by first drawing rectangles, then trapezoids to approximate the area under the curve to discover the physical meanings of the area they have computed. They use Simpson's rule as another approach to approximate the area under a curve.

CONCEPTS	COMMON CORE STATE STANDARDS
Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions	**AP Calc 1.1.** Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
Use graphical calculators to verify and estimate limits	**AP Calc 1.2.** Students use graphical calculators to verify and estimate limits.
Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function	**AP Calc 2.0**. Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
Demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem	**AP Calc 3.0.** Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.
Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function	**AP Calc 4.1.** Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

Understand the interpretation of the derivative as an instantaneous rate of change. Use derivatives to solve a variety of problems that involve the rate of change of a function	**AP Calc 4.2.** Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
Use differentiation to sketch, by hand, graphs of functions. Identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing	**AP Calc 9.0.** Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
Introduce the definition of the definite integral by using Riemann sums to approximate integrals.	**AP Calc 13.0.** Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
Introduce Simpson's rule as another approach to approximate the area under the curve.	**AP Calc 21.0.** Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.
MATHEMATICAL PRACTICES	
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

★**Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.**
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Derivatives can be interpreted as rates of change in various situations • Derivatives of a function gives information about the original function • The concept of a limit is one of the foundations of calculus. • The limit of a function is the value approached by f(x) as x approaches a given value or infinity. • The derivative is the instantaneous rate of change at a given point. • Derivatives can be used to solve a variety o f problems involving instantaneous rate of change. • Limits can be determined using algebra, graphs and/or tables of data.	1) What strategies can be applied to determine the limit of a polynomial? 2) What relationship exists between the local maximum and minimum locations on a graph and a limit or derivative? 3) How does the derivative represent an instantaneous rate of change? 4) How does the integral represent the summation of an infinite set? 5) How do you determine that a function is continuous and/or differentiable? 6) Is there a way to visualize what a derivative is? 7) How can the concept of limits be applied in mathematics? 8) How is the concept of a limit connected to a derivative ? 9) What is the best method to use to find the limit of a function? 10) How do limits approaching infinity help describe the asymptotic behavior of a function? 11) How do limits help determine the continuity of a function?	• Rate of change • Velocity • Instantaneous rate of change • Average rate of change • Derivative • Asymptotes • Open interval • Close interval • Approximation • Area under the curve • Concavity • Difference quotient • Displacement • Acceleration

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp **KHAN Academy** **https://www.khanacademy.org** **KHAN Academy – Limits** **https://www.khanacademy.org/math/differential-calculus/limits_topic** **Derivatives & Rates of Change** **http://math.njit.edu/docs/C2_6M139SelfAssessment.pdf** **Ms. Roshan's Libriary** **http://www.screencast.com/users/Ms.Roshan** **AP Central** **http://apcentral.collegeboard.com** **Larson's Calculus** **http://hmco.tdlc.com/public/icalc/** **Visual Calculus** **http://archives.math.utk.edu/visual.calculus/** **Approximating the area under a curve:**	**Cooperative learning**: Engage all students by using a variety of differentiation strategies including but not limited to questioning techniques, wait time, Think-Pair-Share, peer tutoring, small groups collaboration, etc. **Checking for understanding and reflecting on students' background knowledge:** Use a variety of strategies to frequently check for understanding such as small white boards, hand signals (thumbs up/thumbs down), parking lot questions, etc. Teachers make connections to students' prior knowledge **Problem-solving and abstract reasoning:** Analyze the data, compare/contrast, use counterexamples, construct plausible arguments, make conclusions, justify different ways to solve a problem and communicate to others. **Quick write:** Explain the process and the solution by using academic language and key vocabulary **Modeling and solving real-world problems:** Apply algebraic skills and knowledge to solve a variety of engaging/ relevant problems to make assumptions, analyze the data, derive to solutions and draw viable conclusions **Technology-enhanced instruction:** Utilize graphing calculators, spreadsheets, computer algebra systems, statistical packages and other appropriate software **Project-based learning:** Use a variety of problem-solving assignments such as creating/ solving word	Limits: http://www.wilsonareasd.org/wahs/Vitko/AP%20Calculus/2%20Limits%20and%20Continuity/Assessment/ Rate of Change: https://www.math.dartmouth.edu/~klbooksite/2.01/201.html Rate of Change: http://education-portal.com/academy/exam/topic/rate-of-change.html Riemann Sums: http://web.henry.k12.va.us/cms/lib04/VA01000023/Centricity/Domain/389/Riemann_Sums_b.pdf Practice Tests: http://ryono.net/exams_precalch_tests.html

http://www.education.com/study-help/article/rectangular-approximations/ **Paul's online Math Notes:** **http://tutorial.math.lamar.edu/Classes/CalcI/Tangents_Rates.aspx** **Area under a curve:** **https://www3.nd.edu/~apilking/Math10550/Lectures/24.%20Areas%20and%20Distances.pdf**	problems and the culminating unit tasks.	

LANGUAGE GOALS

- Students will describe orally and in writing the purposes of and differences among sample surveys, experiments, and observational studies.
  *Example: "*Based on the survey of teenage high school students, more students are more/less likely to _____ than _____."

- Students will decide whether a specified model is consistent with results from a data simulation.
  Example: "A model stating that a spinning coin falls heads up with a probability of 0.5 is not consistent with a simulation result of 5 tails in a row."

- Students will explain orally and in writing how they use statistical and probability concept in their lives, using the following specific set of words: *distribution, mean, standard deviation, probability,* and *statistics.*
  Example: "Based on the distribution of test scores with a mean of _____ and a standard deviation of _____, a test score of _____ is (*adjective).*
- Students will explain orally and in writing areas under the normal curve allow us to answer and model real life situations.

PERFORMANCE TASKS

Modeling:

https://www.math.dartmouth.edu/~klbooksite/2.01/201.html

http://www.cpm.org/pdfs/information/sampleChapters/PCT_Ch9_TV.pdf

Tasks/Activities:

http://illuminations.nctm.org/Lesson.aspx?id=2955

http://illuminations.nctm.org/unit.aspx?id=6085

Projects:

http://realteachingmeansreallearning.blogspot.com/p/open-ended-math-projects-and-lessons.html

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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
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