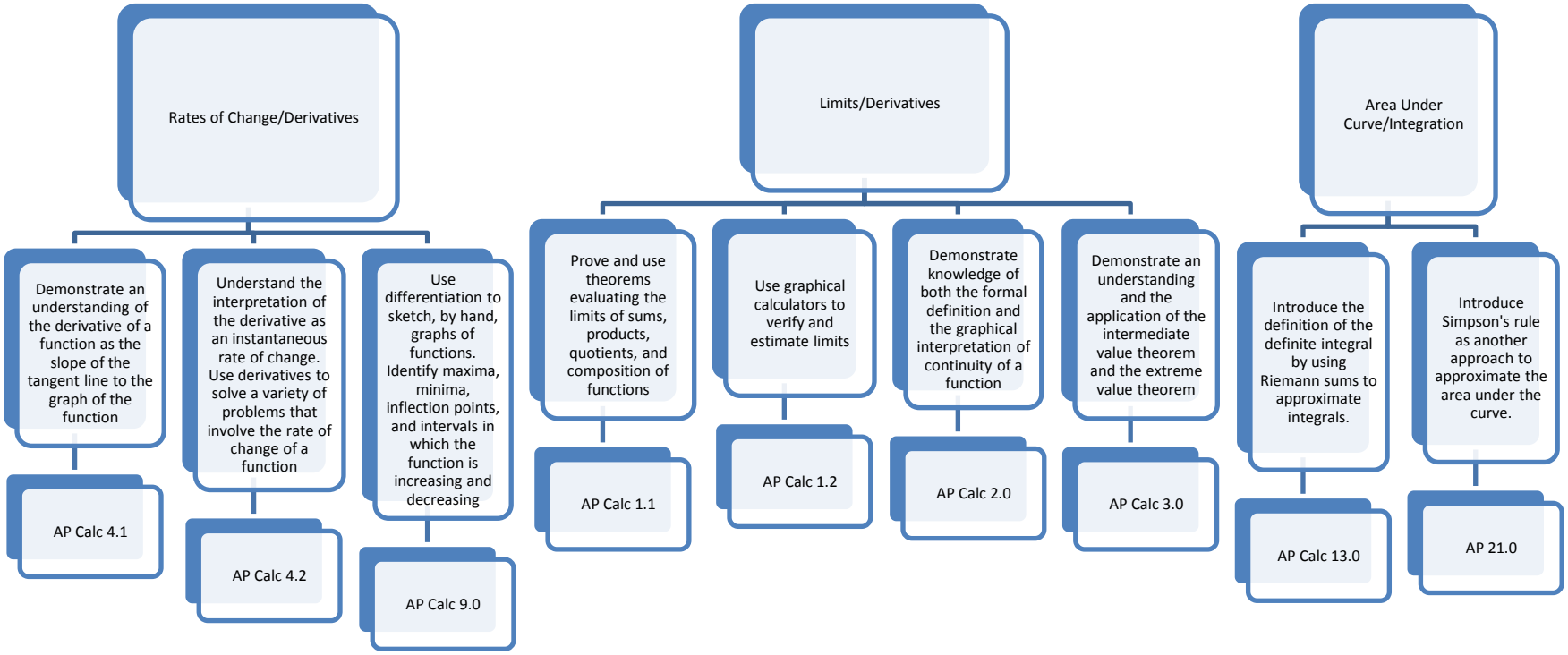
Honors Advanced Mathematics

Unit 6

Introduction to Calculus



Honors Advanced Math – UNIT 6
Introduction to Calculus

Critical Area:
Students investigate average rate of change and focus on the numeric analysis of change over a short intervals of time, leading to discussion of limit, and instantaneous rate of change. Students engage in intuitive understanding of limiting process. They calculate limits using algebra. Given the correct notation, students practice and apply the definition of limit with various kinds of functions, including piece-wise defined functions with a step-discontinuity. Students formally define limit, and practice existence proofs of limits as x approaches a fixed number. Students justify answers analytically, graphically, numerically, and verbally and construct viable argument regarding the non-routine problems posed. Students understand the interplay between the geometric and analytic information and use calculus to predict and to explain the observed local and global behavior of a function. Students numerically analyze curves by first drawing rectangles, then trapezoids to approximate the area under the curve to discover the physical meanings of the area they have computed. They use Simpson's rule as another approach to approximate the area under a curve.

CONCEPTS	COMMON CORE STATE STANDARDS
Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions	**AP Calc 1.1.** Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
Use graphical calculators to verify and estimate limits	**AP Calc 1.2.** Students use graphical calculators to verify and estimate limits.
Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function	**AP Calc 2.0**. Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
Demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem	**AP Calc 3.0.** Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.
Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function	**AP Calc 4.1.** Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

Understand the interpretation of the derivative as an instantaneous rate of change. Use derivatives to solve a variety of problems that involve the rate of change of a function	**AP Calc 4.2.** Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
Use differentiation to sketch, by hand, graphs of functions. Identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing	**AP Calc 9.0.** Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
Introduce the definition of the definite integral by using Riemann sums to approximate integrals.	**AP Calc 13.0.** Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
Introduce Simpson's rule as another approach to approximate the area under the curve.	**AP Calc 21.0.** Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning.**	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Derivatives can be interpreted as rates of change in various situations • Derivatives of a function gives information about the original function • The concept of a limit is one of the foundations of calculus. • The limit of a function is the value approached by f(x) as x approaches a given value or infinity. • The derivative is the instantaneous rate of change at a given point. • Derivatives can be used to solve a variety of problems involving instantaneous rate of change. • Limits can be determined using algebra, graphs and/or tables of data.	1) What strategies can be applied to determine the limit of a polynomial? 2) What relationship exists between the local maximum and minimum locations on a graph and a limit or derivative? 3) How does the derivative represent an instantaneous rate of change? 4) How does the integral represent the summation of an infinite set? 5) How do you determine that a function is continuous and/or differentiable? 6) Is there a way to visualize what a derivative is? 7) How can the concept of limits be applied in mathematics? 8) How is the concept of a limit connected to a derivative ? 9) What is the best method to use to find the limit of a function? 10) How do limits approaching infinity help describe the asymptotic behavior of a function? 11) How do limits help determine the continuity of a function?	• Rate of change • Velocity • Instantaneous rate of change • Average rate of change • Derivative • Asymptotes • Open interval • Close interval • Approximation • Area under the curve • Concavity • Difference quotient • Displacement • Acceleration

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp **KHAN Academy** **https://www.khanacademy.org** **KHAN Academy – Limits** **https://www.khanacademy.org/math/differential-calculus/limits_topic** **Derivatives & Rates of Change** **http://math.njit.edu/docs/C2_6M139SelfAssessment.pdf** **Ms. Roshan's Libriary** **http://www.screencast.com/users/Ms.Roshan** **AP Central** **http://apcentral.collegeboard.com** **Larson's Calculus** **http://hmco.tdlc.com/public/icalc/** **Visual Calculus** **http://archives.math.utk.edu/visual.calculus/** **Approximating the area under a curve:**	**Cooperative learning**: Engage all students by using a variety of differentiation strategies including but not limited to questioning techniques, wait time, Think-Pair-Share, peer tutoring, small groups collaboration, etc. **Checking for understanding and reflecting on students' background knowledge:** Use a variety of strategies to frequently check for understanding such as small white boards, hand signals (thumbs up/thumbs down), parking lot questions, etc. Teachers make connections to students' prior knowledge **Problem-solving and abstract reasoning:** Analyze the data, compare/contrast, use counterexamples, construct plausible arguments, make conclusions, justify different ways to solve a problem and communicate to others. **Quick write:** Explain the process and the solution by using academic language and key vocabulary **Modeling and solving real-world problems:** Apply algebraic skills and knowledge to solve a variety of engaging/ relevant problems to make assumptions, analyze the data, derive to solutions and draw viable conclusions **Technology-enhanced instruction:** Utilize graphing calculators, spreadsheets, computer algebra systems, statistical packages and other appropriate software **Project-based learning:** Use a variety of problem-solving assignments such as creating/ solving word	Limits: http://www.wilsonareasd.org/wahs/Vitko/AP%20Calculus/2%20Limits%20and%20Continuity/Assessment/ Rate of Change: https://www.math.dartmouth.edu/~klbooksite/2.01/201.html Rate of Change: http://education-portal.com/academy/exam/topic/rate-of-change.html Riemann Sums: http://web.henry.k12.va.us/cms/lib04/VA01000023/Centricity/Domain/389/Riemann_Sums_b.pdf Practice Tests: http://ryono.net/exams_precalch_tests.html

http://www.education.com/study-help/article/rectangular-approximations/ **Paul's online Math Notes:** **http://tutorial.math.lamar.edu/Classes/CalcI/Tangents_Rates.aspx** **Area under a curve:** **https://www3.nd.edu/~apilking/Math10550/Lectures/24.%20Areas%20and%20Distances.pdf**	problems and the culminating unit tasks.	

LANGUAGE GOALS

- Students will describe orally and in writing the purposes of and differences among sample surveys, experiments, and observational studies.
  *Example: "*Based on the survey of teenage high school students, more students are more/less likely to _____ than _____."

- Students will decide whether a specified model is consistent with results from a data simulation.
  Example: "A model stating that a spinning coin falls heads up with a probability of 0.5 is not consistent with a simulation result of 5 tails in a row."

- Students will explain orally and in writing how they use statistical and probability concept in their lives, using the following specific set of words: *distribution, mean, standard deviation, probability,* and *statistics.*
  Example: "Based on the distribution of test scores with a mean of _____ and a standard deviation of _____, a test score of _____ is (*adjective).*
- Students will explain orally and in writing areas under the normal curve allow us to answer and model real life situations.

PERFORMANCE TASKS

Modeling:

https://www.math.dartmouth.edu/~klbooksite/2.01/201.html

http://www.cpm.org/pdfs/information/sampleChapters/PCT_Ch9_TV.pdf

Tasks/Activities:

http://illuminations.nctm.org/Lesson.aspx?id=2955

http://illuminations.nctm.org/unit.aspx?id=6085

Projects:

http://realteachingmeansreallearning.blogspot.com/p/open-ended-math-projects-and-lessons.html

References:

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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
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