Honors Advanced Mathematics
Unit 4
Polar Equations and the Complex Plane

- **Conics**
  - Translate between the geometric description and equation for a conic section
    - G-GPE.3, 3.1
- **Complex Number System**
  - Represent complex numbers and their operations on the complex plane
    - N-CN.4-6
- **Polar Equations**
  - Parametric and Polar Function
    - N-CN.4-5
  - Demonstrate an understanding of functions and equations defined parametrically and graph them
    - F-IF.10
  - Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems
    - F-IF.11

Honors Advanced Math – UNIT 4
Polar Equations and the Complex Plane

Critical Area:
Students derive the equations of ellipses and hyperbolas given foci. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, they use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola as well as graph the equation.

Students investigate the geometry of the complex numbers more fully and connect it to operations with complex numbers. They represent complex numbers on the complex plane in both rectangular and polar form. They calculate the distance between numbers in the complex plane as the modulus of the difference. Students expand the skills involved in working with equations into several areas: trigonometric functions, by setting up and solving equations such as $sin\ 2\theta = ½$; parametric functions by making sense of the equations $x = 2t, y = 3t + 1, 0 \leq t \leq 10$. Students develop conceptual knowledge of functions that set the stage for the learning of other standards in Precalculus. They investigate the relationship between the graphs of sine and cosine as a function of theta and also use the parametric form of the functions where $x(\theta) = cos(\theta)$ and $y(\theta) = sin(\theta)$.

CLUSTERS	COMMON CORE STATE STANDARDS
Translate between the geometric and the equation for a conic section	**Geometry: Expressing Geometric Properties with Equations** **G-GPE.3. (+)** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is consistent. **G-GPE.3.1**. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola, and graph the equation
Represent complex numbers and their operations on the complex plan	**N-CN.4.(+)** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN. 5. (+)** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has *modulus 2 and argument 120°*. **N-CN.6**. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

(m) Interpret functions that arise in applications in terms of the context	**F-IF.10. (+)** Demonstrate an understanding of functions and equations defined parametrically and graph them. CA **F-IF.11. (+)** Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. CA
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning**.	
LEARNING PROGRESSIONS	

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The sum or difference of the distances of the foci from the directrix is consistent. • Graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$ can be circles, parabolas, ellipses, or hyperbolas. • The equations of ellipses and hyperbolas can be derived from the foci. • The relationship between the graph of a complex number and their operations and the conjugation of complex numbers on the	1) What are the geometric characteristics of conics? 2) How do you identify the graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$? 3) How can you graph a complex number in rectangular and polar form? 4) What is the relationship between rectangular and polar form of a complex number? 5) What is the importance of knowing the conjugate of a complex number? 6) In terms of their respective equations, what is the difference between a circle and an ellipse?	circle directrix eccentricity ellipses foci hyperbolas parabola parametric function complex number complex plane real axis

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
complex plane can be understood. • Different types of relationships between quantities can be modeled with different types of functions. • Functions and relations can be represented using polar coordinates. • Functions and equations can be defined parametrically. • All functions have algebraic, numerical, graphical and verbal representations. • Graphs are visual representations of solution sets of equations and inequalities. • Graphs of functions can explain the observed local and global behavior of a function. • Real world situations can be modeled and solved by using various functions	7) What relationships exist between quantities that can be modeled by functions? 8) How can functions and relations be represented using polar coordinates? 9) Why is it important to define functions and equations parametrically? 10) What does it mean to solve equations graphically? 11) What do the domain and the range of a function represent?	imaginary axis magnitude modulus argument conjugate polar form polar coordinates rectangular form parametric equations parametric curves

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
NCTM Illuminations •*Cutting Conics*:G-GPE.3 Students explore and discover conic sections by cutting a cone with a plane. Circles, ellipses, parabolas, and hyperbolas are examined using the Conic Section Explorer tool. Physical manipulatives such as dough can optionally be used as well. http://illuminations.nctm.org/Lesson.aspx?id=2907 *Human Conics:* G-GPE.3 Students use sidewalk chalk and rope to illustrate the locus definitions of ellipses and parabolas. Kinesthetics, teamwork, and problem solving are stressed as students take on the role	Students will explore the conic sections and describe how to cut a cone to create the various conic sections. Separate the class into 6 groups (or a multiple of 6 if your class is large). Assign two conic sections to each group. There are 6 different ways to do this: circle/ellipse, circle/hyperbola, circle/parabola, ellipse/hyperbola, ellipse/parabola, and hyperbola/parabola. Each group should create a poster summarizing what they've learned about their two conic sections and comparing and contrasting them. Students will write a summary of either the ellipse or parabola construction for the benefit of a classmate who has missed the lesson. The	1. Ask students to describe how they discovered how to cut their cones to create each conic section–circles, ellipses, parábolas, hyperbolas. 2. Give students a picture of an ellipse and a parabola with possible foci or directrix indicated. Ask them to use a ruler and right angle measure to determine and explain whether or not the figure is actually the named conic. 3. Using data regarding the distance from the Sun and the orbital periods of other planets, ask students to generate parametric equations for the orbits of the other planets in

of focus, directrix, and point on the conic, and figure out how to construct the shape. http://illuminations.nctm.org/Lesson.aspx?id=3003 *Mars Orbit:* F-IF.10 Students will generate parametric equations to describe the position of planets relative to the Sun; then, they will combine the equations to describe the position of Mars relative to Earth.http://illuminations.nctm.org/Lesson.aspx?id=3980 *Axonometry*: N-CN.4, N-CN.5 http://illuminations.nctm.org/Lesson.aspx?id=4228 Wolfram Demonstration: N-CN.4 http://demonstrations.wolfram.com/ComplexNumber/ Engage New York https://www.engageny.org/sites/default/files/resource/attachments/precalculus-m1-module-overview-and-assessments.pdf CN.4 and N-CN.5 **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition, Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison	summary should include the definition and an explanation of how the drawing technique applies the definition. Afterwards, students can exchange and critique their summary with other students. Given parametric equations, group students and ask them to find the polar equation that will give the same shape as the one obtained with given parametric equations. Afterwards, students will share their explanations in a whole class discussion. Use properties of difference of two squares to find the modulus. Relate the modulus visually using vectors. Graph complex numbers and identify the magnitude of the complex number, the distance of the complex number from the origin, and the direction of the complex number from the origin. Express complex numbers in polar coordinate form and in rectangular form. Have students explore the conic sections and describe how to cut a cone to create various conic sections. Import images of circles from fields from Google Earth into a coordinate grid system and find their equations	the solar system relative to the Earth

Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2ndedition, Foerster, Key Curriculum (2007)		

LANGUAGE GOALS

Writing:

1) Students will explain and justify the process of completing the square to identify whether the quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$ is a ellipse, circle, parabola, or a hyperbola.

Example: I completed the process of completing the square by _____ and found that _____. This means that graph of the quadratic equation is a _______.

2) Students will compare and contrast the differences and similarities between ellipses, circles, parabolas, and hyperbolas.

Example: If the eccentricity of a conic section is _______, than the graph is a _______.

Listening and Speaking:

3. Students will generate class discussions using academic vocabulary related to the rectangular and polar forms of complex numbers.

4) Students will participate in class discussions using specific vocabulary related to transformations and functions.

5) Students will explain and justify(orally) how to graph a function to a partner as well as restating and summarizing their partner's explanation.

Example: First I __________ because ____________, second I ___________ because__________,

Reading:

6. Students will read a word problem and identify the language needed to create an algebraic representation in order to solve the problem.

PERFORMANCE TASKS

Textbook:

Larson, R. and Hostetler, R. (2007). Pre-Calculus with Limits, 5th edition. Boston, New York: Houghton/Mifflin.

Publisher: Houghton Mifflin Company

Authors: Larson, R., Hostetler, R.

Topic: Conics

- Suspension Bridge (Page 742, Problem 62)
- Loran (Page 761, Problem 42)
- Satellite Tracking (Page 798, Problem 58)
- Earthquake: Page 667 #3
- Road Design: Page 669 #94
- Architecture: G-GPE.3Page 678 #47-49
- Navigation: Page 688 #46
- Planetary Motion: Page 727 #55

DIFFERENTIATION📖		
FRONT LOADING	ACCELERATION	INTERVENTION
Introduce students to ellipses and help them understand that conics are like circle and parabolas. Introduce students to the equations and graphs of conics and help them see the relationship between equation and graph. Engage students in an activity that would connect their understanding of conics to the real-world. Illustrate conic sections geometrically as cross-sections of a cone. Have students define conic sections and illustrate it pictorially.	Provide examples of real-world problems that can be modeled by circles, parabolas, and ellipses. Students will write and graph equations in polar form. Students will classify conics from their general equation. Students will use properties of parabolas, ellipses, and hyperbolas to model and solve real-life problems	Have students use calculators or computer software to lessen the computational burden in simplifying and graphing conics. Use hands-on activities to allow students to explore how conics may vary (i.e. Using a string and two thumbtacks, have students explore how to obtain ellipses that are long or narrow) Precalculus intervention should include strategies such as targeted grouping peer and counseling grouping, Use informal techniques frequently during regular class time to gauge student understanding. Use questioning that focuses on student thinking and reasoning to help you monitor your students.

		Incorporate writing activities and group work to observe student thinking and identify misconceptions and gaps in understanding. Have students illustrate concepts using drawings, graphs, and models. Many students who need intervention struggle to learn concepts because they may not be able to grasp abstract concepts. Whenever possible, vary your instructional techniques to include use of models, manipulatives, and technology

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics).* Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
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6. http://map.mathshell.org/materials/index.php..Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
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10. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
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15. 13.Foerster,P. A.(2007). Precalculus with Trigonometry Conceptsand Applications, 2nd edition. Emeryville, CA:Key Curriculum