Domain: Number and Operations—Fractions[4]

Student proficiency with fractions is essential to success in algebra at later grades. In grade three students developed an understanding of fractions as built from unit fractions. A critical area of instruction in grade four is developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

Numbers and Operations—Fractions	4.NF

Extend understanding of fraction equivalence and ordering.

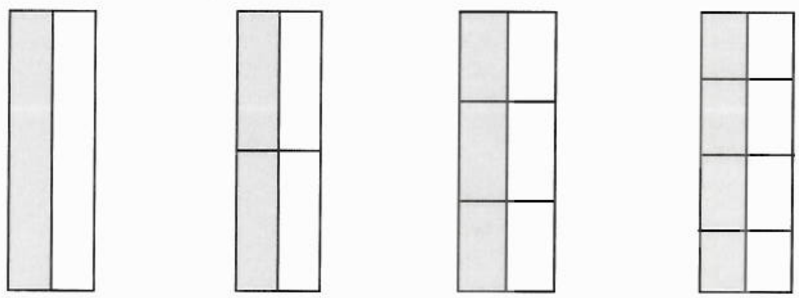
1. Explain why a fraction $a/$ is equivalent to a fraction ($\times a$)/(b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Grade four students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction (e.g., $\frac{a}{b} = \frac{n \times a}{n \times b}$, for $n \neq 0$). Students use visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (**4.NF.1▲**). This property forms the basis for much of the work with fractions in fourth grade; including comparing, adding, and subtracting fractions and the introduction of finite decimals.

Students reason about and explain why fractions are equivalent using visual models. For example, the area models below all show fractions equivalent to $\frac{1}{2}$, and while in grade three students simply justified that all the models represent

[4] In grade four fractions include those with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

the same amount visually, in grade four students reason about *why* it is true that $\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{3\times1}{3\times2}=\frac{4\times1}{4\times2}$, etc. They use reasoning such as: when a horizontal line is drawn through the center of the first model to obtain the second, both the number of equal parts and the number of those parts we are counting double ($2\times2=4$ in the denominator, $2\times1=2$ in the numerator, respectively), but even though there are more parts counted they are smaller parts. Students notice connections between the models and the fractions they represent in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions. Students also emphasize the inversely related changes: the number of unit fractions becomes larger, but the size of the unit fraction becomes smaller.



$\frac{1}{2}$ $\quad \frac{2}{4}=\frac{2\times1}{2\times2}$ $\quad \frac{3}{6}=\frac{3\times1}{3\times2}$ $\quad \frac{4}{8}=\frac{4\times1}{4\times2}$

(Adapted from Arizona 2012)

Students should have repeated opportunities to use pictures such as these and the ones below to understand the general method for finding equivalent fractions. Of course, students may also come to see that the rule works both ways, for example:

$$\frac{28}{35}=\frac{7\times4}{7\times5}=\frac{4}{5}.$$

Teachers must be careful to not overemphasize this “simplifying” of fractions, as there is no mathematical reason for doing so, though depending on the problem context one form may be more desirable. In particular, teachers should avoid the use of the term “reducing” fractions for this process, as the value of the fraction itself is *not* being reduced. A more neutral term such as “renaming” (which hints to these fractions simply being different names for the same amount) allows for referring to this strategy without the potential for student misunderstanding.

[Note: Sidebar]

Focus, Coherence, Rigor:

While it is true that one can justify that $\frac{a}{b} = \frac{n\times a}{n\times b}$ by arguing that:

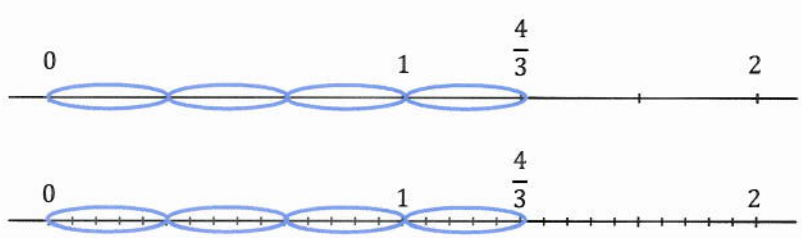
$$\frac{n\times a}{n\times b} = \frac{n}{n}\times\frac{a}{b} = 1\times\frac{a}{b} = \frac{a}{b}$$

i.e., that we are simply multiplying by 1 in the form of $\frac{n}{n}$, since students *have not yet encountered* the general notion of fraction multiplication in fourth grade, this argument should be avoided in favor of developing an understanding with diagrams and reasoning about the size and number of parts that are created in this process. Students will learn the general rule that $\frac{a}{b}\times\frac{c}{d} = \frac{ac}{bd}$ in grade five.

Examples: Reasoning With Diagrams That $\frac{a}{b} = \frac{n\times a}{n\times b}$.

Using an Area Model: The whole is the rectangle, measured by its area. The picture on the left shows the area divided into three rectangles of equal area (thirds) with two of them shaded (2 pieces of size $\frac{1}{3}$), representing $\frac{2}{3}$. On the right, the vertical lines divide the parts (the thirds) into smaller parts. There are now 4×3 smaller rectangles of equal area, and the shaded area now comprises 4×2 of them, so it represents $\frac{4\times 2}{4\times 3}$.

Using a Number Line: The top number line shows $\frac{4}{3}$: it is 4 parts when the unit length is divided into three equal parts and then iterated. When each of the intervals of length $\frac{1}{3}$ is further divided into 5 equal parts, there are now 5×3 of these new equal parts in the unit interval. Since 4 of the $\frac{1}{3}$ parts were circled before, and each of these has been subdivided into 5 parts, there are now 5×4 of these new small parts. Therefore $\frac{4}{3} = \frac{5\times 4}{5\times 3} = \frac{20}{15}$.



(Above examples adapted from Progressions 3-5 NF 2012)

Creating equivalent fractions by dividing and shading squares or circles, and matching each fraction to its location on the number line can reinforce students' understanding of fractions. For example, see "Equivalent Fractions" available at http://illuminations.nctm.org/activitydetail.aspx?id=80 (NCTM Illuminations 2013). Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (**4.NF.2▲**). They compare fractions using benchmark fractions, and by finding common denominators or common numerators. Students explain their reasoning and record their results using >, < and = symbols.

Examples: Comparing Fractions.
1. Students might compare fractions to benchmark fractions, e.g. comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8} = \frac{1}{2}$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6} = \frac{1}{2}$, it must be true that $\frac{3}{8} < \frac{2}{3}$.
2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5\times12}{8\times12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7\times8}{12\times8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. Notice that students do not need to find the smallest common denominator for two fractions; any one will work.
3. Students can also find a common numerator to compare $\frac{5}{8}$ and $\frac{7}{12}$. They find that $\frac{5}{8} = \frac{5\times7}{8\times7} = \frac{35}{56}$ and $\frac{7}{12} = \frac{7\times5}{12\times5} = \frac{35}{60}$. They then reason that since parts of size $\frac{1}{56}$ are larger than parts of size $\frac{1}{60}$ when the whole is the same, that $\frac{5}{8} > \frac{7}{12}$.

Numbers and Operations—Fractions	4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations o whole numbers.

3. Understand a fraction *a*/ with as sum of fractions 1/*b*.
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 1/8 1/8*

1/8 ; 3/8 1/8 2/8 ; 1/8 1/8 8/8 8/8 1/8.

c. Ad and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

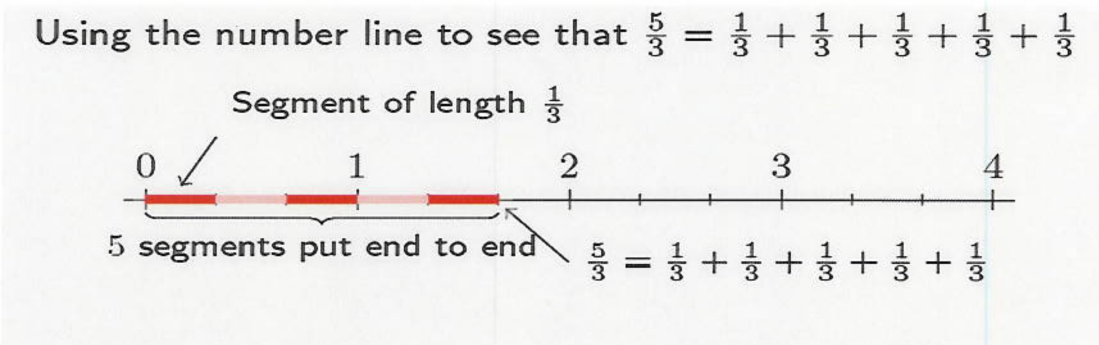
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

In grade four students extend previous understanding of addition and subtraction of whole numbers to add and subtract fractions with like denominators (**4.NF.3▲**). They begin by understanding a fraction $\frac{a}{b}$ as a sum of the unit fractions $\frac{1}{b}$. In grade three, students learned that the fraction $\frac{a}{b}$ represented a parts when a whole is broken into b equal parts (i.e., parts of size $\frac{1}{b}$.) However, in grade four, students connect this understanding of a fraction with the operation of addition; for instance, they see now that if a whole is broken into 4 equal parts and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ (**4.NF.3b▲**). They experience composing fractions from and decomposing fractions into sums of unit fractions and non-unit fractions in this general way, e.g., by seeing $\frac{5}{4}$ also as

- $\frac{1}{4}+\frac{1}{4}+\frac{3}{4}$
- $\frac{2}{4}+\frac{3}{4}$
- $\frac{1}{4}+\frac{3}{4}+\frac{1}{4}$, etc.

Working with this standard supports student learning of **(4.NF.3a▲)** and **(4.NF.3d▲)** by writing and using unit fractions. It also helps students avoid the common misconception of adding two fractions by adding their numerators and denominators, e.g. erroneously writing $\frac{1}{2}+\frac{5}{6}=\frac{6}{8}$. Work with **(4.NF.3b▲)** helps students see that the unit fraction for the total is the same as the unit fractions being added and grouped into fractions made from that unit fraction. In general, the meaning of addition is the same for both fractions and whole numbers. Students understand addition as "putting together" like units and they visualize how fractions are built from unit fractions and that a fraction is a sum of unit fractions.

Students may use visual models to support this understanding, for example, showing that $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ by using a number line model. **(MP.1, MP.2, MP.4, MP.6, MP.7).**



(Source: Progressions 3-5 NF 2012)

Students add or subtract fractions with like denominators, including mixed numbers **(4.NF.3a, c▲)** and solve word problems involving fractions **(4.NF.3d▲).** They connect their understanding of any fraction as being composed of unit fractions to realize that, for example:

$$\frac{7}{5}+\frac{4}{5}=\overbrace{\frac{1}{5}+\cdots+\frac{1}{5}}^{7}+\overbrace{\frac{1}{5}+\cdots+\frac{1}{5}}^{4}=\overbrace{\frac{1}{5}+\cdots+\frac{1}{5}}^{7+4}=\frac{7+4}{5}.$$

This quickly allows students to develop a general principle that $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$. Using similar reasoning, students understand that $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$.

Students also compute sums of whole numbers and fractions, by realizing that any whole number can be written as an equivalent number of unit fractions of a given size, e.g. they find the sum $3+\frac{7}{2}$ in the following way:

$$3+\frac{7}{2}=\frac{6}{2}+\frac{7}{2}=\frac{13}{2}.$$

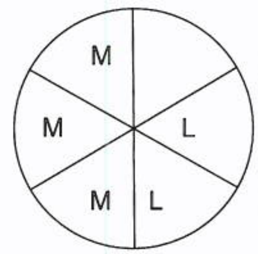
Understanding this method of adding a whole number and fraction allows students to accurately convert mixed numbers into fractions, e.g.:

$$4\frac{5}{8} = 4 + \frac{5}{8} = \frac{32}{8} + \frac{5}{8} = \frac{37}{8}.$$

Students should develop a firm understanding that a mixed number indicates the sum of a whole number and a fraction (i.e., $a\frac{b}{c} = a + \frac{b}{c}$), and should learn a method for converting them to fractions that is connected to the meaning of fractions such as the one above, rather than typical rote methods.

Examples: Reasoning With Addition and Subtraction of Fractions. (4.NF.3a-d▲).

1. Mary and Lacey share a pizza. Mary ate $\frac{3}{6}$ of the pizza and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat altogether? Use the picture of a pizza to explain your answer.

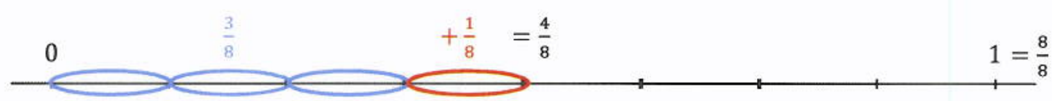


Solution: "I labeled three sixths for Mary and two sixths for Lacey. I can see that altogether they've eaten $\frac{5}{6}$ of the pizza. Also, I know that

$$\frac{3}{6} + \frac{2}{6} = \frac{2+3}{6} = \frac{5}{6}."$$

2. Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Is it enough to complete the packaging?

Solution: "I know I need to find $5\frac{3}{8} + 3\frac{1}{8}$ to find out how much they have altogether. I know that altogether they have $3 + 5 = 8$ feet of ribbon plus the other $\frac{1}{8} + \frac{3}{8}$ feet of ribbon. Altogether this is $8\frac{4}{8}$ feet of ribbon, which means they have enough ribbon to do their packaging. They even have $\frac{1}{8}$ feet of ribbon left."



3. Elena, Matthew, and Kevin painted a wall. Elena painted $\frac{5}{9}$ of the wall and Matthew painted $\frac{3}{9}$ of the wall. Kevin paints the rest. How much of the wall does Kevin paint? Use the picture to help find your answer.

Solution: "I can show in the picture that Elena and Matthew painted $\frac{8}{9}$ altogether by shading what Elena and Matthew painted. The remaining that Kevin paints is $\frac{1}{9}$. I can write this as $1-\frac{8}{9}=\frac{1}{9}$, or even $1-\frac{5}{9}-\frac{3}{9}=\frac{1}{9}$." (New York State Education Department [NYSED] 2012).

Numbers and Operations—Fractions 4.NF

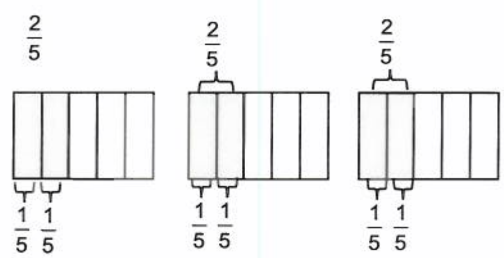
Build fractions from unit fractions by applying and extending previous understandings of operations o whole numbers.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction *a*/ as multiple of 1/*b* *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of *a*/ as multiple of 1/*b* and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of fraction by whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Previously in grade three, students learned that 3 × 7 can be represented as the total number of objects in 3 groups of 7 objects, and that they could find this by finding the sum 7 + 7 + 7. Grade four students apply this concept to fractions, understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (**4.NF.4a▲**). Intimately connected with standard **(4.NF.3)**, students make the shift to seeing $\frac{5}{3}$ as $5\times\frac{1}{3}$, for example by seeing:

$$\frac{5}{3}=\overbrace{\frac{1}{3}+\cdots+\frac{1}{3}}^{\text{5 times}}=5\times\frac{1}{3}$$

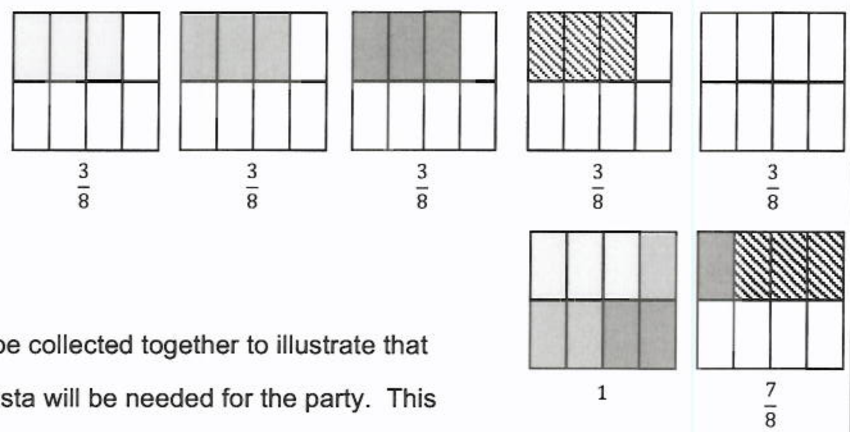
Students then extend this understanding to make meaning of the product of a whole number and a fraction (**4.NF.4b▲**), for example, by seeing $3\times\frac{2}{5}$ as:



$$\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{3\times2}{5}=\frac{6}{5}.$$

(Progressions 3-5 NF 2012)

Students are presented with opportunities to work with problems involving multiplication of a fraction by a whole number in context to relate situations, models, and corresponding equations (**4.NF.4c▲**).

Example: Multiplying a Fraction by a Whole Number (4.NF.4c▲).
Each person at a dinner party eats $\frac{3}{8}$ of a pound of pasta. There are 5 people at the party. How many pounds of pasta are needed? Pasta comes in 1-lb boxes. How many boxes should be bought? *Solution*: If five rectangles are drawn, with $\frac{3}{8}$ of a pound shaded in each rectangle, then students see that they are finding $5\times\frac{3}{8}=\frac{15}{8}$.  The separate eighths can be collected together to illustrate that altogether $1\frac{7}{8}$ pounds of pasta will be needed for the party. This means that 2 boxes should be bought.

(Adapted from Arizona 2012 and N. Carolina 2011)

Numbers and Operations—Fractions 4.NF
Understand decimal notation for fractions, and compare decimal fractions. 5. Express fraction with denominator 1 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 4/100 34/100.* 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe length as 0.62 meters; locate 0.62 o number line diagram.* 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

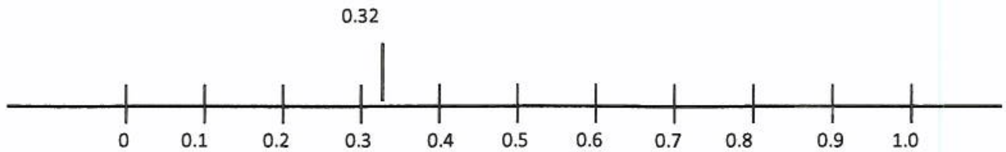
In fourth grade students develop an understanding of decimal notation for fractions and compare *decimal fractions* (fractions with denominator 10 or 100). This work lays the foundation for performing operations with decimal numbers in grade five. Students learn to add decimal fractions by converting them to fractions with the same denominator (**4.NF.5▲**). For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100}+\frac{4}{100}=\frac{34}{100}$. Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and 100 (Adapted from Progressions 3-5 NF 2012).

In grade four, students first use decimal notation for fractions with denominators 10 or 100 (**4.NF.6▲**), understanding that the number of digits to the right of the decimal point indicates the number of zeros in the denominator. Students make connections between fractions with denominators of 10 and 100 and place value. They read and write decimal fractions; for example, students say 0.32 as "thirty-two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

Focus, Coherence, Rigor.

Teachers are urged to consistently use place value based language when naming decimals to reinforce student understanding, i.e., by saying "four tenths" when referring to 0.4, as opposed to "point four", and by saying "sixty eight hundredths" when referring to 0.68, as opposed to "point sixty eight" or "point six eight."

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. Students reason that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$, so it would be placed on the number line near that value. **(MP.2, MP.4, MP.5, MP.7)**



Students compare two decimals to hundredths by reasoning about their size (**4.NF.7▲**).They relate their understanding of the place value system for whole numbers to fractional parts represented as decimals. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator and that the "wholes" are the same.

Common misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that $0.2 < 0.07$ simply because $2 < 7$.
- Students sometimes think the longer the decimal number the greater the value. For example they think that 0.03 is greater than 0.3.