1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

   Research Says:
   - Learning progressions or trajectories describe how students make transitions from prior knowledge to more sophisticated understandings
   - Both teachers and students need to be able to answer these crucial questions:
     - What mathematics is being learned?
     - Why is this important?
     - How does it relate to what has already been learned?
     - Where are these mathematical ideas going?
   - Situating learning goals within the mathematical landscape supports opportunities to:
     - Build explicit connections
     - See how ideas build and relate to one another
     - Develop a coherent and connected view of the discipline

Establish mathematics goals to focus learning Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

   Research Says:
   - Effective math teaching and learning uses carefully selected tasks as one way to motivate student learning and build new knowledge.
   - Research on math tasks over the past two decades has found:
     - Not all tasks provide the same opportunities for student thinking and learning.
     - Student learning is the greatest in classrooms where tasks consistently encourage high-level student thinking and the least in classrooms where tasks are routinely procedural in nature.
     - Tasks with high cognitive demands are the most difficult to implement well and are often transformed into less demanding tasks.
   - To ensure that students have the opportunity to engage in high-level thinking, teachers must regularly select and implement tasks the promote reasoning and problem solving.

Implement tasks that promote reasoning and problem solving **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple entry points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as needed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

3. **Use and connect mathematical representations.**
Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Research Says:

- Effective mathematics teaching includes a strong focus on using varied mathematical representations.
- Using a variety of representations helps students examine a concept through more than one lens. Selected representations could include:
    - Visual representations
    - Physical representations
    - Symbolic representations
    - Contextual representations
    - Verbal representations
- When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving skills.

(Fuson, Kalchman, & Bransford, 2005; Lesh, Post, and Behr, 1987)

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

   Research Says:
   - Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class.
   - Smith and Stein (2011) describe five practices for effectively using student responses in class discussions:
     - Anticipating student responses prior to the lesson
     - Monitoring students' work on engagement with tasks
     - Selecting particular students to present their mathematical work
     - Sequencing students' responses in specific order for discussion
     - Connecting different students' responses and connecting responses to key mathematical ideas
   - Students must have opportunities to talk with, respond to, and question one another as part of the discourse community, in ways that support the mathematics learning for all students in class.

Facilitate meaningful mathematical discourse **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

5. **Pose purposeful questions.**
Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Research Says:

- Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful discourse.
- Commonalities exist across a number of questioning frameworks. Key cross-cutting aspects of these questioning frameworks include:
    - Gathering information
        - Students recall facts, definitions, or procedures
    - Probing thinking
        - Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completing of a task
    - Making the mathematics visible
        - Students discuss mathematical structures and make connections among mathematical ideas and relationships
    - Encouraging reflection and justification
        - Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.

Pose purposeful questions **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

   Research Says:
   - Effective mathematics teaching focuses on the development of both conceptual understanding and procedural fluency.
   - Both NCTM and CCSS-M emphasize that procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving.
   - Students who use math effectively do much more than carry out procedures. Such students must also know:
     - Which procedure is appropriate and most productive for a given situation,
     - What a given procedure accomplishes, and
     - What kind of results to expect
   - "Mechanical execution of procedures without understanding their conceptual basis often leads to bizarre results" (Martin, (2009), p.165)

Build procedural fluency from conceptual understanding **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.

7. **Support productive struggle in learning mathematics.**
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Research Says:

- Effective mathematics instruction supports students in struggling *productively* as they learn mathematics.
- Teacher actions to support students in productive struggle include:
    - Students engage in problems that take time to solve
        - Teachers select tasks that promote reasoning and problem solving; explicitly encouraging students to persevere; finding ways to support students without removing challenges in a task.
    - Students explain and discuss how they thought about and solved tasks
        - Teachers ask students to explain and justify how they solved a task, and value the quality of the explanation as much as the final solution.
    - Students have a responsibility and obligation to make sense of the math
        - Teachers give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.
    - Students use important tools in making sense of the task
        - Teachers give students access to tools that will support their thinking process.
    - Students communicate one's thinking during a task
        - Teachers ask students to explain their thinking and pose questions based on students' reasoning, rather than on the way the teacher is think about the task.

Support productive struggle in learning mathematics **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

8. **Elicit and use evidence of student thinking.**
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Research Says:

- Effective mathematics teaching elicits evidence of student's current mathematical understanding and uses it as the basis for making instructional decisions.
- A focus on evidence includes:
    - Identifying indicators of what is important to notice in students' mathematical thinking
    - Planning for ways to elicit that information
    - Interpreting what the evidence means with respect to students' learning
    - Deciding how to respond on the basis of students' understanding
- Using assessment *for* learning means that:
    - Students are revealing their mathematical understanding, reasoning, and methods in classroom discourse and written work.
    - Students reflect on mistakes and misconceptions to improve their understanding
    - Students ask questions, responding to, and giving suggestions to support the learning of their classmates
    - Students assess and monitor their own progress towards math learning goals, and identify areas they can improve.

Elicit and use evidence of student thinking **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Identifying what counts as evidence of student progress toward mathematics learning goals. Eliciting and gathering evidence of student understanding at strategic points during instruction. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. Reflecting on mistakes and misconceptions to improve their mathematical understanding. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.